

Three Types of Equisingularity for Curves

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Introduction. There are three ways in which two plane complex algebraic or algebroid curve branches at the origin can be compared, algebraically, by looking at their fractional power-series, topologically, by looking at their embeddings as subspaces of the plane, and, also topologically, by looking at the knots they cut on a small sphere about the origin. The object of this note is to show that all three comparisons are equivalent.

The relation between fractional power series and knots is of course a classical result (*cf.* [4] p. 13). But the treatment here, connecting both of these notions with the idea of topological equisingularity, appears to give a very elegant elementary approach to the whole question. It is also an approach which gives some promise of extensions to higher dimensions.

1. Definitions and the main theorem. Let C be an algebraic or algebroid curve branch given in a neighborhood of the origin of the complex plane by the equation $f(x, y) = 0$, a polynomial in y with coefficients which are polynomials, or convergent power series in x . It will be assumed that f is irreducible as a power series in x and y over the complex numbers. But f has roots of the form $y = \phi(x)$, where ϕ is a fractional power series in x , actually a power series in $x^{1/r}$ for some integer r . If $\phi(x)$ is written as $\sum a_i x^{v_i}$, then each non-zero term $a_i x^{v_i}$ in which a new factor appears in the denominator of the exponent is called a *characteristic term* and the corresponding exponent is a *characteristic exponent*. Two fractional power series which have the same characteristic exponents are said to be of the *same species*. For example if

$$\phi(x) = x^{1/2} + x^{3/2} + x^{5/2} + x^{17/6} + x^{19/6}$$

the characteristic exponents are $1/2$ and $17/6$, corresponding to the introduction of first the factor 2 and then the factor 3 on the denominator. This series has the same species as $x^{1/2} - x^{17/6}$. Of course since f is assumed to be irreducible, as a power series in x and y it follows that all its roots, fractional power series in x , will be of the same species. They are in fact conjugate to each other and are obtained from any one of them by multiplying x by a suitable root of unity. Thus this common species can be spoken of as the species of the curve branch. Two branches will be called *algebraically equisingular* if they have the same species.