

On the Existence of Shock Curves of Quasilinear Wave Equations

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1. Introduction and notation. We consider the problem $y_{tt} - Q^2(y_x)y_{xx} = 0$: $y(x, 0) = 0$, $y_t(x, 0) = y_0(x)$, $0 \leq x \leq L$; $y(0, t) = y(L, t) = 0$, $t \geq 0$, where $Q(\xi)$ and $y_0(x)$ satisfy the conditions proposed by MacCamy and Mizel [8]. They discussed the problem in three cases and showed that a local C^2 solution exists and eventually breaks down in the sense that some of its second order derivatives become unbounded at a finite time. This system is not genuinely nonlinear in the sense explained later. We consider only the second case of [8] and impose one additional assumption that $Q(\xi)$ and $y_0(x)$ are analytic. We show that any break-down point, where the break-down occurs, there originates a unique shock curve. The methods used in this paper are applicable to parts of the first case and of the third case of [8]. The author is grateful to Professor Hans F. Weinberger for many stimulating conversations.

Let $u(x, t) = y_x(x, t)$ and $v(x, t) = y_t(x, t)$. The problem is then

$$(1.1) \quad \begin{aligned} u_t - v_x &= 0, \\ v_t - Q^2(u)u_x &= 0, \end{aligned}$$

$$(1.2) \quad \begin{aligned} u(x, 0) &= 0, & v(x, 0) &= v_0(x), & 0 \leq x \leq L, \\ v(0, t) &= v(L, t) = 0, & t &\geq 0. \end{aligned}$$

We assume that Q is a positive analytic function and

$$(1.3) \quad Q'(\xi) > 0 \text{ for } \xi < 0 \text{ and } Q'(\xi) < 0 \text{ for } \xi > 0.$$

We assume that $v_0(x) = f|_{[0, L]}(x)$ is concave over $[0, L]$, where $f(x)$ is an odd periodic analytic function over $(-\infty, \infty)$ with period $2L$ and $f(0) = f(L) = 0$.

We denote $\max_{0 \leq x \leq L} f(x)$ by $a = f(b)$, $0 < b < L$. We define

$$(1.4) \quad M(\xi) = \int_0^\xi Q(\zeta) d\zeta, \quad a_1 = M(\infty), \quad a_2 = -M(-\infty).$$

We assume further that

$$(15) \quad a > \max(a_1, a_2).$$