

A Method for Computing Non-parameteric Area Minimizing Surfaces over n Dimensional Domains, Together with a priori Error Estimates

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0. Introduction. This paper treats the problem of determining, computationally, a non-parametric minimal surface with given boundary values. In particular we work under the following hypotheses: $\Omega \subset \mathbf{R}^n$ ($n \geq 2$) is non-empty, bounded, open, and convex and $u : \text{Bdry } (\Omega) \rightarrow \mathbf{R}$ satisfies the bounded slope condition with constant M . Denote by $U : \text{Clos } (\Omega) \rightarrow \mathbf{R}$ the continuous function the graph of which minimizes \mathcal{H}^n measure subject to agreeing with u on $\text{Bdry } (\Omega)$. We show how, for each $\delta > 0$ and $\epsilon > 0$, to obtain a function F , piecewise linear over a triangulation of \mathbf{R}^n into pieces congruent to

$$\{x : 0 \leq x_1 \leq x_2 \leq \cdots \leq x_n \leq \delta\},$$

such that

$$\sup \{|F(x) - U(x)| : x \in \text{Clos } (\Omega)\} \leq c_{17} \delta^{1/6(n+1)} + \epsilon.$$

Here c_{17} depends only on n , M , $\mathcal{L}^n(\Omega)$, and $\text{diam } (\Omega)$ and is explicitly defined in 1 (9).

1. Definitions and notation. Except when otherwise stated, we shall follow the notation and terminology of [1]. Throughout this paper, n will denote a fixed integer, $n \geq 2$, Ω will denote a fixed, non-empty, bounded, convex subset of \mathbf{R}^n , and

$$u : \text{Bdry } (\Omega) \rightarrow \mathbf{R}$$

will denote a fixed function satisfying the bounded slope condition with constant M (see [3, Definition 1.1]).

(1) Set

$$A = \mathcal{L}^n(\Omega)$$

and