

Interpolation by the Real Method between BMO, $L^\alpha(0 < \alpha < \infty)$ and $H^\alpha(0 < \alpha < \infty)$

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0. Preliminaries. Suppose A_0 and A_1 are quasi-normed spaces continuously embedded in a topological vector space A . In the real method of interpolation, spaces intermediate between A_0 and A_1 are defined by means of an interpolating function $K(t, f; A_0, A_1)$. If $f \in A_0 + A_1$, define

$$K(t, f; A_0, A_1) = \inf_{f=f_0+f_1} (\|f_0\|_{A_0} + t \|f_1\|_{A_1})$$

where the infimum is taken over all choices of f_0 and f_1 such that $f_0 \in A_0$, $f_1 \in A_1$ and $f = f_0 + f_1$. The intermediate space $(A_0, A_1)_{\theta, q}$ is then defined as the space of all functions f in $A_0 + A_1$ such that

$$\|f\|_{(A_0, A_1)_{\theta, q}} = \left(\int_0^\infty [t^{-\theta} K(t, f; A_0, A_1)]^q \frac{dt}{t} \right)^{1/q} < \infty$$

where $0 < \theta < 1$ and $0 < q \leq \infty$. A complete treatment is given by Sagher [10] and Peetre and Sparr [8].

The space $BMO(\mathbb{R}^n)$ may be defined as those functions f such that

$$\|f\|_{BMO} = \sup_Q \left\{ \inf_c \left(\frac{1}{|Q|} \int_Q |f - c| \right) \right\} < \infty$$

where the supremum is taken over all cubes Q in \mathbb{R}^n , and the infimum is taken over all real values c . In the following results the underlying space will always be \mathbb{R}^n , although any measure space with a regular Vitali family would suffice (see Riviere [9]). We will extend previous results to identify $(L^\alpha, BMO)_{\theta, q}$ and $(H^\alpha, BMO)_{\theta, q}$ for $0 < \alpha < \infty$ and show the equivalence of BMO with the more generally defined family of spaces BMO^r for $0 < r < \infty$.

1. The spaces $(L^\alpha, BMO)_{\theta, q}$ ($0 < \alpha < \infty$). The spaces $(L^\alpha, L^\infty)_{\theta, q}$ with $0 < \alpha < \infty$ have been identified by Krée [7] as

$$(L^\alpha, L^\infty)_{\theta, q} = L^{p_\alpha}; \quad p_\alpha = \frac{\alpha}{1 - \theta}$$

where $0 < \theta < 1$, $0 < q \leq \infty$ and L^{p_α} is the Lorentz pq -space. This result is