

# *The Extended Variational Method Applied to the Bifurcation of Solutions to Nonlinear Problems*

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**Introduction.** We propose to study the solution sets of nonlinear eigenvalue problems associated with a class of variational operators in a real separable Hilbert space. In particular we will show the existence of solution branches bifurcating from the isolated eigenvalues of finite multiplicity of the bounded self-adjoint linearization of a not necessarily completely continuous operator. We use a variational approach similar to that used by Berger [2] in proving the existence of bifurcation points for a completely continuous potential operator.

For our work we assume to begin with that  $A$  and  $B$  are odd variational operators defined in  $H$  of the forms  $A = L + C$  and  $B = I + D$  where  $L$  is a bounded self-adjoint operator,  $I$  is the identity,  $C$  and  $D$  are both compact and uniformly continuous on bounded sets, and where there are two continuous nondecreasing functions  $\gamma : \mathbf{R}^+ \rightarrow \mathbf{R}^+$  and  $\delta : \mathbf{R}^+ \rightarrow \mathbf{R}^+$  for which both  $\gamma(r)/r \rightarrow 0$  and  $\delta(r)/r \rightarrow 0$  as  $r \rightarrow 0$  so that  $\|C(u)\| \leq \gamma(\|u\|)$  and  $\|D(u)\| \leq \delta(\|u\|)$ . Furthermore it is assumed that there is some  $\beta_0 > -1$  so that  $\langle Du, u \rangle \geq \beta_0 \|u\|^2$  for all  $u \in H$ . For convenience, we assume that  $\beta_0 < 0$ .

By *variational operators* we mean that  $A$  and  $B$  are the gradients of the functions  $f$  and  $\phi$  respectively where

$$(1) \quad \begin{aligned} f(u) &= \frac{1}{2} \langle Lu, u \rangle + \int_0^1 \langle C(su), u \rangle ds \quad \text{and} \\ \phi(u) &= \frac{1}{2} \|u\|^2 + \int_0^1 \langle D(su), u \rangle ds. \end{aligned}$$

By  $A$  and  $B$  odd we mean that both  $A(-u) = -A(u)$  and  $B(-u) = -B(u)$  for all  $u \in H$ .

The eigenvalue problem which we consider is to find the solution pairs  $(\lambda, u) \in \mathbf{R} \times H$  to

$$(N) \quad Au = \lambda Bu.$$

The linearization of this problem is