

# *Partial Differential Equations, Orlicz Spaces, and Measure Functions*

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**1. Introduction.** We shall call  $h$  a measure function if  $h$  is a continuous nondecreasing function on the interval  $[0, \infty)$  with  $h(0) = 0$  and  $h(t) > 0$  for  $t > 0$ . Then as is well known, [2, p. 6],  $h$  gives rise to a Hausdorff measure  $\Lambda_h$  which, in particular, is defined (in the extended real number system) for all Borel sets  $Z \subset \Omega$  where  $\Omega$  is a bounded open set in Euclidean  $n$ -space,  $\mathbb{R}^n$ ,  $n \geq 2$ . In this paper, we shall establish a theorem connecting measure functions, exceptional sets for partial differential equations, and various Orlicz spaces.

With respect to Orlicz spaces, we shall adopt a combination of the notation used in [1] and in [4]. We shall call  $A$  an  $N$ -function if  $A(t) = \int_0^t \alpha(s) ds$  for  $0 \leq t < \infty$  where  $\alpha$  is a right continuous nondecreasing function on  $[0, \infty)$  with  $\alpha(0) = 0$ ,  $\alpha(t) > 0$  for  $t > 0$ , and  $\lim_{t \rightarrow \infty} \alpha(t) = \infty$  (see [1, p. 228] or [4, p. 6]). If  $\alpha^-(s) = \sup_{\alpha(t) \leq s} t$  and  $A^-(t) = \int_0^t \alpha^-(s) ds$ , then  $A$  and  $A^-$  will be called complementary  $N$ -functions. As is well known complementary  $N$ -functions satisfy Young's inequality, *i.e.*, for  $0 \leq s, t$ ,

$$(1.1) \quad st \leq A(t) + A^-(s).$$

The class  $K_A(\Omega)$  will be the set of all measurable functions  $u$  satisfying  $\int_{\Omega} A(|u(x)|) dx < \infty$ .  $L_A(\Omega)$  will be the smallest vector space under pointwise addition and scalar multiplication which contains  $K_A(\Omega)$ .

In  $L_A(\Omega)$ , we introduce the following norm:

$$(1.2) \quad \|u\|_{(A)} = \inf \left\{ k > 0 : \int_{\Omega} A(|u(x)|/k) dx \leq 1 \right\}.$$

In [1], it is shown that  $L_A(\Omega)$  is a Banach space with respect to the norm (1.2).

Bearing in mind that in this paper  $\Omega$  is a bounded open subset of  $\mathbb{R}^n$ , we define  $E_A(\Omega)$  to be the closure in the Banach space  $L_A(\Omega)$  of the space of functions  $L^\infty(\Omega)$ . It will be the space of functions  $E_A(\Omega)$  which will be of primary