

Duality and Perturbations of Φ_+ and Φ_- -Operators

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1. Introduction and notations. It is a well-known result of Gokhberg, Krein [2] and Kato [4] that if T is a semi-Fredholm operator and P a bounded operator of norm small enough or a compact operator, acting between Banach spaces, then $T + P$ is a semi-Fredholm operator with the same index as T . In the case of small perturbations, Kato gives a precise upper bound of the norm of P : $\|P\| < \gamma(T)$, where $\gamma(T) = \|\hat{T}^{-1}\|^{-1}$ if $T \neq 0$ ($\gamma(T) = \infty$ otherwise), \hat{T} being the canonical one-to-one operator induced by T . In geometric terms, this means that there exist positive constants λ and μ such that $TB \supset \lambda B' \cap R(T)$, $PB \subset \mu B'$ and $\mu < \lambda$, B, B' being the unit balls of the domain space and the range space.

Some results concerning small perturbations of Φ_- -operators in more general locally convex spaces are given by Vladimirskii in [11] (see also [5]), but they do not fully cover the precise Kato's theorem in the case of Banach spaces.

Using duality and Kato's theorem in the Banach spaces generated by closed equicontinuous sets, we prove some results, announced in [7], which constitute a full extension of Kato's theorem on small perturbations of Φ_- -operators and a refinement of several of the above-mentioned results in [11], [5] (another possible extension of Kato's theorem is given in [1]).

We also prove that if T is a Φ_+ -operator, P a small bounded or precompact operator, and either the domain space or the range of T is complete, then $T + P$ is a Φ_+ -operator with the same index as T . This result, for continuous Φ_+ -operator T , has been given without proof in [13].

In the sequel, E, F shall always denote two locally convex Hausdorff topological vector spaces and T, P two (linear) operators from E into F such that $[D(T)]^- \subset D(P)$, where $[D(T)]^-$ is the closure of the domain of definition of T . By *neighborhood* we mean an absolutely convex neighborhood of the origin. We denote by $\langle B \rangle, \rangle B \langle$ respectively the absolutely convex hull and the linear hull of a set B . A *disk* is an absolutely convex set. A disk B is a *Banach* (respectively *finite*) *disk* if the space generated by B , i.e. $\rangle B \langle$ topologized by the Minkowski gauge of B , is a Banach (respectively euclidean) space.

We denote by $N(T)$ and $R(T)$ the kernel and the range of T . The operator T is said to be *open* (respectively *almost open*) if, for any neighborhood U in E , TU