

# *A Free Boundary Problem: The Flow of Two Immiscible Fluids in a One-Dimensional Porous Medium, I*

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**1. Introduction.** This paper establishes a local existence theorem for a one-dimensional free boundary problem suggested by the equations modelling the flow of two slightly compressible, immiscible fluids in a porous medium. Two liquids with known pressure distributions are assumed to be initially in contact at a given point  $s_0$ , and the problem is then to determine the new pressure distributions of each fluid and the new point of contact at any later time. The equations to be satisfied are these:

$$(1.1) \quad u_t(x, t) = \begin{cases} a u_{xx}(x, t), & 0 < x < s(t), 0 < t < T \\ b u_{xx}(x, t), & s(t) < x < 1, 0 < t < T, \end{cases}$$

$$(1.2) \quad \begin{aligned} u(0, t) &= f_1(t), & 0 \leq t \leq T, \\ u(1, t) &= f_2(t), & 0 \leq t \leq T, \\ u(x, 0) &= u_0(x), & 0 \leq x \leq 1, \end{aligned}$$

$$(1.3) \quad u^-(s(t), t) = u^+(s(t), t), \quad 0 \leq t \leq T,$$

$$(1.4) \quad a u_x^-(s(t), t) = b u_x^+(s(t), t), \quad 0 \leq t \leq T,$$

$$(1.5) \quad s(0) = s_0,$$

$$(1.6) \quad \dot{s}(t) = -a u_x^-(s(t), t), \quad 0 \leq t \leq T.$$

Here  $T > 0$ ,  $1 > s_0 > 0$ ,  $a, b, f_1, f_2$ , and  $u_0$  are given; the unknowns are  $s(t)$  and  $u(x, t)$ . (The superscripts  $+$  and  $-$  indicate the limits from right and left respectively for the space variable  $x$ .)

**Remark 1.1.** Problem (1.1)–(1.6) is a one-dimensional and parabolic version of a free boundary problem proposed by Muskat [12]. ■