

Eigenvalues and Stacking Methods in Ergodic Theory

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1. Introduction. The main points of this paper are to develop general results concerning eigenvalues of transformations produced by a stacking method introduced in [7], and to apply these results to construct transformations of various spectral types.

Stacking methods which are discussed by Friedman [5] were developed as a source for examples. Von Neumann and Kakutani produced ergodic measure preserving transformations with discrete spectrum whose eigenvalues are all roots of unity. Chacon [2], using a stacking method, produced transformations with 1 as their only eigenvalue. Christiansen [4] generalized the method of Chacon. The procedure discussed here generalizes all of these techniques.

The objective of the first three theorems is to produce a formula for the set Λ_T of all eigenvalues of T . This facilitates the proofs of the remaining theorems which are existence theorems. For an arbitrary subgroup G of the unit circle group K , Halmos [6] constructs a measure space (Y, β, u) and a transformation T such that T has discrete spectrum with $\Lambda_T = G$. Since eigenvalues of ergodic transformations are simple, the structure of $L^2(Y)$ is of a trivial nature if G is a finite group. Here ergodic measure preserving transformations T on (X, \mathcal{A}, m) where X is the unit interval and m the Lebesgue measure are constructed with $\Lambda_T = G$ where G is an arbitrary finite subgroup of K . Hence such a T does not have discrete spectrum but is of a mixed spectral type. Also, a transformation is constructed having an arbitrary non-root of unity as an eigenvalue and no roots of unity as eigenvalues. Thus, an example of a completely ergodic transformation which is not weakly mixing is produced via a stacking procedure.

2. Preliminaries. Let T be an invertible measure preserving map from X onto X . T is *ergodic* if $m(A) > 0$ and $T A \subset A$ implies $m(A) = 1$, $A \in \mathcal{A}$. T is *completely ergodic* if T^k is ergodic for each positive integer k . Let U_T be the induced unitary operator on $L^2(X)$ defined by $U_T f(x) = f(T(x))$. The eigenvalues of U_T are sometimes referred to as the eigenvalues of T . It can be shown if T is ergodic but not completely, then there is an eigenvalue λ of T such that $\lambda \neq 1$ and $\lambda^k = 1$ for some positive integer k . An ergodic transformation T is *weakly*