

Operator Algebras of Finite Strict Multiplicity. II

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Introduction. This is a supplement to the author's articles [6], [7]. Let $\mathcal{L}(\mathcal{X})$ denote the algebra of all (bounded linear) operators acting on the complex Banach space \mathcal{X} and let \mathcal{A} be a weakly closed, identity containing) subalgebra of $\mathcal{L}(\mathcal{X})$; the *strict multiplicity* $\bar{\mu}(\mathcal{A})$ of \mathcal{A} is the infimum of all cardinal numbers $\mathfrak{c}(\Gamma)$ taken over all the subsets $\Gamma \subset \mathcal{X}$ such that $\mathcal{A}(\Gamma) =$ (definition) *linear span* $\{Ax : A \in \mathcal{A}, x \in \Gamma\}$ coincides with \mathcal{X} . If $\bar{\mu}(\mathcal{A}) < \infty$, we shall say that \mathcal{A} is an *algebra of finite strict multiplicity* (a.f.s.m.). In particular, \mathcal{A} is called *strictly cyclic* if $\bar{\mu}(\mathcal{A}) = 1$, i.e., whenever $\mathcal{X} = \mathcal{A}(\{x_0\})$ for a suitable (strictly cyclic) vector x_0 ; if, moreover, $Ax_0 = 0, A \in \mathcal{A}$ implies that $A = 0$, then we shall say that x_0 separates points of \mathcal{A} .

As in [9], the four weakly closed algebras naturally associated with an operator $T \in \mathcal{L}(\mathcal{X})$ will be denoted by $\mathcal{A}(T)$, $\mathcal{A}^a(T)$, $\mathcal{A}'(T)$ and $\mathcal{A}''(T)$ (the algebra generated by the polynomials in T , the algebra generated by the rational functions in T with poles outside the spectrum $\sigma(T)$ of T , the commutant and the double commutant, respectively). We shall say that T is strictly cyclic if $\mathcal{A}(T)$ is so.

The notation, *etc.*, follows in every sense that of [7] and this article will be used as a general reference throughout the paper. Also, we number the sections and results of the present paper following the numbering of [7]. In the first part (§ 7) the following problem is considered: What can be said about the point spectra $\sigma_p(T)$, $\sigma_p(T^*)$ of T and its adjoint T^* if one of the algebras associated with T has finite strict multiplicity? If $\mathcal{A}''(T)$ is a strictly cyclic algebra, what can be said about its Gelfand spectrum and its relation with $\sigma(T)$?

The second part deals with quasisimilarity and invariant subspaces. Among other results it is shown that an operator A quasisimilar to a T such that $\bar{\mu}(\mathcal{A}''(T)) < \infty$ is necessarily similar to T . The paper includes many examples showing that the results contained here (and several related results) are essentially sharp. Indeed, the counterexamples are more important than the positive results. As usual, *subspace* will always denote a closed linear manifold.

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