

Harmonizability, V-Boundedness and Stationary Dilation of Stochastic Processes

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The main purpose of this paper is to show that a stochastic process X_t , t in the real line R , with values in a given complex Hilbert space \mathcal{H} is harmonizable if and only if it is the projection of some stationary process taking values in a larger Hilbert space \mathcal{K} . Harmonizable processes are natural generalizations of stationary processes where the inner product (X_t, X_s) is continuous and depends only on $t - s$. It is well known that any stationary process X_t , $t \in R$, can be represented in the form

$$X_t = \int_R e^{-itu} d\Phi(u),$$

where Φ is a countably additive orthogonally scattered \mathcal{H} -valued measure on R such that the spectral measure F defined by

$$F(A \cap B) = (\Phi(A), \Phi(B)), \quad A \text{ and } B \text{ Borel sets}$$

is a bounded countably additive nonnegative measure (for the definition of orthogonally scattered measures, see [9]). In this case the correlation function $R(t, s) = (X_t, X_s)$ has the representation

$$R(t, s) = \int_R e^{-i(t-s)u} dF(u).$$

The notion of harmonizable processes was introduced by M. Loève [8] as those processes X_t for which

$$X_t = \int_R e^{-itu} d\Phi(u)$$

and

$$R(t, s) = \iint_{R^2} e^{-i(tv - su)} dF(u, v),$$

with Φ being a countably additive \mathcal{H} -valued measure (not necessarily orthogonally scattered) and the spectral measure F being a complex-valued measure of