

A Free Boundary Problem: The Flow of Two Immiscible Fluids in a One- Dimensional Porous Medium II

LAWRENCE C. EVANS

1. Introduction. In this paper we continue an investigation, begun in [5], of the free boundary problem:

$$(1.1) \quad u_t(x, t) = \begin{cases} a u_{xx}(x, t) & 0 < x < s(t), 0 < t < T \\ b u_{xx}(x, t) & s(t) < x < 1, 0 < t < T \end{cases}$$

$$(1.2) \quad \begin{aligned} u(0, t) &= f_1(t) & 0 \leq t \leq T \\ u(1, t) &= f_2(t) & 0 \leq t \leq T \\ u(x, 0) &= u_0(x) & 0 \leq x \leq 1 \end{aligned}$$

$$(1.3) \quad u^-(s(t), t) = u^+(s(t), t) \quad 0 \leq t \leq T$$

$$(1.4) \quad au_x^-(s(t), t) = bu_x^+(s(t), t) \quad 0 \leq t \leq T$$

$$(1.5) \quad s(0) = s_0$$

$$(1.6) \quad \dot{s}(t) = -au_x^-(s(t), t) \quad 0 \leq t \leq T.$$

Here $T > 0$, $1 > s_0 > 0$, $a > 0$, and $b > 0$ are given constants; f_1 , f_2 , and u_0 are given functions; and the unknowns are $s(t)$ and $u(x, t)$. (The superscripts $+$ and $-$ denote the limits from the right and left respectively for the space variable x .)

Equations (1.1)–(1.6) form a simplified model for the behavior of two slightly compressible, immiscible fluids within a porous medium. The function $u(x, t)$ represents the fluid pressure, and $s(t)$ is the point of contact between the two liquids at time $t \geq 0$. Conditions (1.3) and (1.4) are assumptions that both the pressures and velocities of the fluids are continuous across the interface; (1.6) is the observation that the rate of change of $s(t)$ is the common velocity of the liquids at that point.

The author's previous paper [5] on problem (1.1)–(1.6) was devoted primarily to establishing a *local* existence theorem. We proved that there exists some $T > 0$, depending on the H^1 -norm of the initial data u_0 , such that (1.1)–(1.6) has at least one solution $\{u(x, t), s(t)\}$ for $0 \leq t \leq T$. The techniques of [5] were not,