

Continuous Functions of Operators: A Functional Calculus

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1. Decomposable functions. In [5] the author introduced an operator-valued spectrum for operators on a separable Hilbert space that parallels the usual spectrum of a normal operator. This paper develops a functional calculus that parallels the fact that when T is a normal operator, the von Neumann algebra generated by T is the set of bounded Borel functions of T , and the C^* -algebra generated by T is the set of continuous functions of T . A key ingredient is the consideration of a class of functions whose natural domain is a set of operators rather than scalars. These functions should prove useful in studying and comparing singly generated C^* -algebras.

In [2] and [4] A. Brown, C. K. Fong, and D. W. Hadwin, motivated by Borel functions of normal operators, introduced the class of *decomposable functions* for the purpose of developing a general theory of “parts” of operators (generalizing such concepts as “normal part”). Continuous decomposable functions were used by the author [4] in studying the closures of unitary equivalence classes of operators. Decomposable functions are precisely the functions that will be used in the functional calculus of this paper.

Throughout, H denotes a separable, infinite-dimensional Hilbert space and $B(H)$ denotes the set of *operators* (bounded linear transformations) on H . Also “ $M \leq H$ ” denotes “ M is a subspace of H ”. If $T \in B(H)$, then $C^*(T)$ denotes the C^* -algebra generated by T , 1, and $W^*(T)$ denotes the von Neumann algebra generated by T .

Definition 1.1. A *decomposable function* on H is a function $\varphi : \cup \{B(M) : M \leq H\} \rightarrow \cup \{B(M) : M \leq H\}$ such that

- (a) $\varphi(B(M)) \subseteq B(M)$ whenever $M \leq H$,
- (b) if $T \in B(H)$, $M \leq H$, and M reduces T , then M reduces $\varphi(T)$ and $\varphi([T]_M) = \varphi(T)|_M$,
- (c) if $M, N \leq H$ and $S \in B(M)$ and $U : N \rightarrow M$ is unitary, then $\varphi(U^*SU) = U^*\varphi(S)U$.

A decomposable function φ is (norm) *continuous* if $\varphi|_{B(M)}$ is continuous whenever $M \leq H$ (equivalently, if $\varphi|_{B(H)}$ is continuous).