

# *Asymptotic Behavior of the Principal Eigenfunction for a Singularly Perturbed Dirichlet Problem*

ALLEN DEVINATZ & AVNER FRIEDMAN

**1. Introduction.** Consider a singularly perturbed Dirichlet problem

$$(1.1) \quad \frac{\epsilon}{2} \sum_{j,k=1}^n a_{jk}(x) \frac{\partial^2 u}{\partial x_j \partial x_k} + \sum_{i=1}^n b_i(x) \frac{\partial u}{\partial x_i} = -\lambda_\epsilon u \quad \text{in } D, \quad \epsilon > 0,$$

$$u = 0 \quad \text{on } \partial D.$$

In the papers [1][3][6][7][8] asymptotic estimates were obtained for the principal eigenvalue  $\lambda_\epsilon$  as  $\epsilon \rightarrow 0$ . In this paper we are concerned with the behavior as  $\epsilon \rightarrow 0$  of the principal eigenfunction  $u = u_\epsilon$  when normalized by

$$(1.2) \quad \int_D u_\epsilon^2(x) dx = 1;$$

recall that  $u_\epsilon$  is positive throughout  $D$ . It will be shown that the asymptotic behavior of  $u_\epsilon$  depends on the vector  $b = (b_1, \dots, b_n)$  and, more specifically, on the behavior of the solutions of the ordinary differential system

$$(1.3) \quad \frac{d\xi}{dt} = b(\xi), \quad \xi(0) \in D.$$

In Section 2 we prove that if all the trajectories of (1.3) are attracted toward a single point  $x_0 \in D$ , then the  $u_\epsilon$  converge to a constant and, indeed, uniformly on compact subsets of  $D$ . In Section 3 it is shown that if  $x_0$  is a ‘‘globally repulsive’’ equilibrium point for (1.3), then  $u_\epsilon^2$  converges to the Dirac delta function supported at  $x_0$ .

In case the trajectories starting at any point of  $D$  leave  $\bar{D}$  in finite time, we have been able to analyze the situation only in the case of ordinary differential equations. In Section 4 we prove that, in this case,  $u_\epsilon^2$  converges to the Dirac delta function supported at one end point of  $D$ , depending on the direction of  $b$ .

Finally, in Section 5 we extend the results of Section 2, in the case of ordinary differential equations, to the situation where the equation (1.3) has more than one stable equilibrium point.