

Generalized Hopf Bifurcation and Perturbation in a Full Neighborhood of a Given Vector Field

NATHANIEL CHAFEE

Introduction. We shall consider a differential equation

$$(1) \quad \dot{x}(t) = f_0(x(t))$$

where $x(t)$ is a real n -vector and f_0 is a C^∞ -smooth function defined on some open neighborhood of the origin in R^n . We shall assume that the origin $x_0 = 0$ is an equilibrium point for (1), that is, $f_0(0) = 0$. Also, we shall assume that the Jacobian matrix $f'_0(0)$ has a complex conjugate pair of simple eigenvalues $\pm i$, and we shall assume that any other eigenvalue λ of $f'_0(0)$ is not an integer multiple of λ , that is, $\lambda \neq mi$ for each $m = 0, \pm 1, \pm 2, \dots$.

Let us refer to (1) as the unperturbed differential equation and, correspondingly, let us refer to f_0 as the unperturbed vector field.

By the perturbed differential equation we shall mean an equation of the form

$$(2) \quad \dot{x}(t) = f(x(t))$$

where f is C^∞ -smooth on some open neighborhood of the origin in R^n and where, with respect to an appropriate topology, f is close to f_0 . We can refer to f itself as the perturbed vector field.

We are interested in the existence of nonzero periodic orbits Γ for (2) with Γ lying near the origin $x_0 = 0$ in R^n and with Γ having period T close to 2π . The question we ask is, how many such orbits Γ does Eq. (2) actually have? With reference to a suitable function space for the elements f appearing in (2), we want to answer the question for each f belonging to some open neighborhood \mathcal{N} of f_0 .

Let us phrase this problem in another way. For any small neighborhood \mathcal{N} of f_0 , and for each integer $j \geq 0$, let S_j denote the set of all $f \in \mathcal{N}$ such that Eq. (2) has exactly j nonzero periodic orbits Γ of the type required. Our problem is, which of the sets S_j are nonempty? Clearly, the answer must depend on f_0 . We want to see exactly what that dependence is.

All of the above constitutes the problem we want to solve. Our solution of that problem is the subject of this paper.