

Selfadjointness of Second Order Degenerate-Elliptic Operators

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1. In this paper we shall discuss the question of selfadjointness for the formally selfadjoint differential operator

$$(1.1) \quad H = - \sum_{j,k} \partial_j a_{jk}(x) \partial_k + q(x), \quad x \in R^n.$$

When the operator H is strongly elliptic there is a large literature devoted to this question and relatively complete results are available. However, when H is degenerate elliptic, *i.e.*, when the matrix $(a_{jk}(x))$ is only positive semi-definite, then little seems to be known. In [1] we obtained some results for degenerate operators with matrix coefficients by applying A. E. Nussbaum's theory of quasi-analytic vectors. In this paper we shall get somewhat better results for numerical coefficients by the use of somewhat more sophisticated techniques, relying heavily on the theory of stochastic differential equations.

2. We shall suppose that $(a_{jk}(x))$ is a real, semi-definite, symmetric matrix in $C^1(R^n)$ and that $q \in L^2_{loc}(R^n)$. Thus H acts on $L^2(R^n)$ with values in the set of distributions of order ≤ 2 . If we set $H'_0 = H|_{C^\infty_0(R^n)}$, and let H_0 be its closure, then H_0 is a symmetric operator in L^2 . We shall call this the minimal operator associated with H . If $u \in D(H_0^*)$, then it is immediate that $H_0^*u = Hu$, and thus H_0^* is the restriction of H to those $u \in L^2$ for which $Hu \in L^2$.

For the remainder of this paper we shall make the following more restrictive assumptions on the a_{jk} .

$$(2.1) \quad a_{jk} \in C^2(R^n) \text{ with bounded second derivatives.}$$

The assumption (2.1) implies that

$$(2.2) \quad \begin{aligned} a_{jk}(x) &= 0(1 + |x|^2), \\ \nabla a_{jk}(x) &= 0(1 + |x|). \end{aligned}$$

The hypothesis (2.1) implies, by a theorem of Freidlin [3], that

$$(2.3) \quad (a_{jk}(x)) = \frac{1}{2} \sigma^2(x),$$