

# *Hyperinvariant Subspaces of the Direct Sum of Certain Contractions*

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Let  $T$  be a bounded linear operator acting on a complex separable Hilbert space  $H$ . A subspace  $K$  of  $H$  is said to be hyperinvariant for  $T$  if  $K$  is invariant for every operator that commutes with  $T$ . The hyperinvariant subspaces for various classes of operators have been determined before (cf. [3] and [4]). Recently, there has been some research work in characterizing hyperinvariant subspaces for  $C_0(N)$  contractions [8] and for certain  $C_0$  contractions [9]. In this paper we consider the same problem for other classes of contractions. We characterize the hyperinvariant subspaces for  $T_1 \oplus T_2$ , where, for  $j = 1, 2$ ,  $T_j$  is a completely non-unitary contraction with a scalar-valued characteristic function  $\varphi$ .

Recall that a contraction  $T$  ( $\|T\| \leq 1$ ) is completely non-unitary (c.n.u.) if there exists no non-trivial reducing subspace on which  $T$  is a unitary operator. For any contraction  $T$  on  $H$  there is defined a characteristic function  $\Theta_T(e^{it})$ , where for each  $e^{it}$ ,  $\Theta_T(e^{it})$  is a contraction from  $\mathcal{D}_T \equiv \overline{(I - T^*T)^{1/2}H}$  to  $\mathcal{D}_{T^*} \equiv \overline{(I - TT^*)^{1/2}H}$ . For properties relating the contraction to its characteristic function readers are referred to [6]. If  $\dim \mathcal{D}_T = \dim \mathcal{D}_{T^*} = 1$ , then we denote the scalar-valued characteristic function by  $\varphi_T$ . Let us consider the functional model for such a contraction  $T$ , that is, we consider  $T$  being defined on the space  $H \equiv [H^2 \oplus \Delta \bar{L}^2] \ominus \{(\varphi_T w, \Delta w) : w \in H^2\}$  by

$$T(f, g) = P(e^{it}f, e^{it}g) \quad \text{for } (f, g) \in H,$$

where  $P$  denotes the (orthogonal) projection onto  $H$  and  $\Delta = (1 - |\varphi_T|^2)^{1/2}$ . It was shown in [6] that there is a one-to-one correspondence between the invariant subspaces for  $T$  and the regular factorizations  $\varphi_T = \varphi_2 \varphi_1$  of its characteristic function  $\varphi_T$ . Recall that  $\varphi_T = \varphi_2 \varphi_1$  is called a scalar regular factorization if both  $\varphi_1$  and  $\varphi_2$  are scalar-valued functions and  $|\varphi_2(e^{it})| = 1$  or  $|\varphi_1(e^{it})| = 1$  for almost all  $t$ ; it is called a vector regular factorization if  $\varphi_2 = (f_{21} f_{22})$  and  $\varphi_1 = \begin{pmatrix} f_{11} \\ f_{12} \end{pmatrix}$ , where  $f_{11}, f_{12}, f_{21}$  and  $f_{22}$  are functions in  $H^\infty$  satisfying  $|f_{21}(e^{it})|^2 + |f_{22}(e^{it})|^2 = |f_{11}(e^{it})|^2 + |f_{12}(e^{it})|^2 = 1$  a.e. A regular factorization of  $\varphi_T$  can only be a scalar regular factorization or a vector regular factorization (cf. [6], p. 301). Let  $\varphi_i$  and  $\varphi_e$  denote the inner part and outer part of  $\varphi_T$ , respectively.