

Kähler Metrics of Negative Curvature, the Bergmann Metric near the Boundary, and the Kobayashi Metric on Smooth Bounded Strictly Pseudoconvex Sets

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§1 Introduction. The purpose of this paper is to prove that if $D \subset \mathbb{C}^n$ is a bounded strictly pseudoconvex domain with smooth boundary, then, near the boundary of D , the curvature tensor of the Bergmann metric of D approaches the curvature tensor of constant holomorphic sectional curvature $-4/(n+1)$, and the curvature tensor of the Kähler metric $g_{a\bar{b}} = -\frac{\partial^2}{\partial z_a \partial \bar{z}_b} (\log \psi(z))$, defined near the boundary of D , with $\psi(z)$ the distance from z to the boundary of D , approaches the curvature tensor of constant holomorphic sectional curvature -4 . Thus, both of these metrics have strictly negative Riemannian sectional curvature near the boundary of D . The first of these results is proved by a computation based on Fefferman's asymptotic formula for the Bergmann kernel [2], but does not require an understanding of how Fefferman's formula is obtained. The second result is a corollary of the proof of the first result which does not require Fefferman's formula and is therefore completely elementary. These results will be proved in Section 2. We note that the Bergmann metric of the unit ball has constant holomorphic sectional curvature $-4/(n+1)$, and since near the boundary of D the metrics considered are comparable to the Bergmann metric of the unit ball, our results are not unexpected.

These results have several important, but elementary, corollaries. First, as was pointed out to the author by H. Wu, either result (in particular the second completely elementary result) implies that D admits a complete Kähler metric with all holomorphic sectional curvatures $\leq -c^2 < 0$, and this in turn implies a result of Graham [3], that D is complete hyperbolic. In addition, we obtain Graham's precise estimates on the boundary behavior of the Kobayashi metric. This approach has two advantages: it provides a completely elementary proof of Graham's results on the Kobayashi metric of a bounded strictly pseudoconvex domain with smooth boundary in \mathbb{C}^n and; it allows us to easily general-