

Local Times for Gaussian Vector Fields

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Section 1. Introduction. If $\{X(t) \in R^d : t \in R^N\}$ is a measurable random vector valued field and $\mu(dt) \geq 0$ is a finite measure supported on a compact set $I \subset R^N$, the sojourn time of $X(t)$ in B is

$$\Phi(\omega, B) = \mu\{s \in I : X(s, \omega) \in B\}.$$

If $\Phi(\omega, dx)$ is absolutely continuous with respect to Lebesgue measure then $\varphi(\omega, x) = \Phi(\omega, dx)/dx$ is called the local time of $X(t)$.

This paper treats the problem of finding useful sufficient conditions for the existence and continuity of local times $\varphi(\omega, x)$ for certain Gaussian vector fields and in particular for Lévy's multiparameter Brownian motion, $X(t) = (X_1(t), \dots, X_d(t))$, with independent components for which

$$EX_i(t)X_i(s) = \frac{1}{2} \{|t| + |s| - |t - s|\}.$$

A typical result, see Section 7, is that for $\mu(dt) = dt$ and $d < 2N$ local times $\varphi(\omega, x)$ exist for Lévy's process and are continuous in x with probability 1.

In section 2 we fix some terminology and the main results are described.

In sections 3 and 4 general results are proved about the existence and regularity properties of local times for general (= non-Gaussian) fields. These sections (as well as Sections 5 and 6) derive in large part from the work of Berman [1, 2, 3, 4], where special cases and different versions of much of this can be found. Several new details have been added, but perhaps the major additions to the theory here are the generality and the real variable techniques similar to those of Orey [10] and Marcus [9], as opposed to the Fourier analytic methods of Berman, (see also Kahane [8], ch. 13).

In Sections 5 and 6 the earlier results are applied to Gaussian fields. The major (for us) obstacle here is that Berman's work required the "time parameter" t to be one-dimensional. To our surprise we discovered that Berman's concept of "local nondeterminism" has a useful extension to higher dimensions. Once the right definition is found Berman's scalar results [4] are easily extended to vector valued fields.

In Section 7 we show that Lévy's multiparameter Brownian motion and certain related processes are "locally non-deterministic" and in appropriate dimensions have continuous local times.