

Helson Sets Not Containing the Identity Are Uniform Fatou-Zygmund Sets

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0. Introduction. Let E be a closed set in a locally compact abelian group G . $C_0(E)$ denotes the set of continuous functions on E vanishing at ∞ . A function f on E is *hermitian* if $f(x) = \overline{f(x^{-1})}$ for all $x \in E \cap E^{-1}$. Let

$$C_0^h(E) = \{f \in C_0(E) : f \text{ is hermitian}\}$$

$$A(E) = \{f|_E : f \in A(G)\}$$

$$A_+(E) = \{f|_E : f \in A(G), f \text{ is positive definite}\}$$

E is a *Helson set* if $C_0(E) = A(E)$. If E is a Helson set, then

$$h(E) = \sup \{\|f\|_{A(E)} : f \in C_0(E), \|f\|_{C_0(E)} \leq 1\}$$

is a finite number that is called *the Helson constant of E* . If E is a Helson set, we refer to the number

$$h_0(E) = \sup \{\|f\|_{A(E)} : f \in C_0^h(E), \|f\|_{C_0(E)} \leq 1\}$$

as *the hermitian Helson constant of E* . Helson sets in discrete G are called *Sidon sets*.

E is a *Fatou-Zygmund set* if $C_0^h(E) = A_+(E)$. Let $\|f\|_{A_+(E)} = \inf \{\|g\|_A : g|_E = f, g \text{ is positive definite}\}$, $\|\cdot\|_{A_+(E)}$ is in general not a norm; for instance, $\|1\|_{A_+(G)} \neq \|-1\|_{A_+(G)}$. If E is a $F - Z$ set, then

$$fz(E) = \sup \{\|f\|_{A_+(E)} : f \in C_0^h(E), \|f\|_{C_0(E)} \leq 1\}$$

is a finite number [5, p. 94], that is called *the Fatou-Zygmund constant of E* .

E is a *uniform Helson set* if for every closed set F disjoint from E , every $\varphi \in C_0(E)$ and every $\epsilon > 0$ there exist a constant λ depending only on ϵ and a $g \in A(G)$ satisfying:

$$g|_E = \varphi$$

$$|g| < \epsilon \text{ on } F$$

$$\|g\|_A \leq \lambda \|\varphi\|_{C_0(E)}.$$