

Vector Fields on V -Manifolds, and Locally Free $G \times R^1$ -Actions on Manifolds

CHAO-CHU LIANG

1. We say that a Lie group H acts locally freely on a space X if all the isotropy groups H_x for $x \in X$ are discrete subgroups of H , that is, all the orbits are of the same dimension. Let R^1 denote the additive group of real numbers, and S^1 the circle group. It follows from the Borel formula [5], [6, p. 164], that $S^1 \times S^1$ cannot act locally freely on an odd dimensional sphere S^{2n+1} . In this note, we will show that there does not exist a differentiable locally free $S^1 \times R^1$ -action on S^{2n+1} . This result can be proved directly by applying the argument in [2] to a transversal elliptic operator as in [3, p. 84]. But here we will derive it from the following theorem stated in a slightly more general setting.

Theorem. *If a compact orientable V -manifold (see [4], [11], [12]) M admits a nowhere zero vector field, then its Euler characteristic vanishes.*

2. We recall here briefly the definition of V -manifold given in [4] and [11]. By a local uniformizing system (ℓ .u.s.) $\{U, G, \varphi\}$ for an open subset W of a paracompact Hausdorff space M , we mean a collection of the following objects:

- (1) U : an open disk around the origin in R^n .
- (2) G : a finite group of orthogonal transformation of U .
- (3) φ : a continuous map of U onto W such that $\varphi \circ g = \varphi$ for all $g \in G$, and φ induces a homeomorphism from U/G onto W .

Let $\{U, G, \varphi\}$ and $\{U', G', \varphi'\}$ be ℓ .u.s.'s for W and W' respectively such that $W \subseteq W'$. By an injection of $\{U, G, \varphi\}$ into $\{U', G', \varphi'\}$ we mean a diffeomorphism λ of U into U' such that there exists an isomorphism η of G into G' satisfying $\lambda \circ g = \eta(g) \circ \lambda$ and $\varphi = \varphi' \circ \lambda$.

A $C^\infty V$ -manifold consists of a connected paracompact Hausdorff space M and a family \mathcal{F} of ℓ .u.s.'s for open subsets of M satisfying the following conditions:

- (1) If $\{U, G, \varphi\}, \{U', G', \varphi'\} \in \mathcal{F}$, and $W = \varphi(U)$ is contained in $W' = \varphi'(U')$, then there exists an injection of $\{U, G, \varphi\}$ into $\{U', G', \varphi'\}$.
- (2) The open sets W , for which there exists a ℓ .u.s. $\{U, G, \varphi\} \in \mathcal{F}$, form a basis of open sets in M .