

The Endomorphisms of L_p ($0 \leq p \leq 1$)

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1. Introduction. In [12] Kwapien gives a representation theorem for the endomorphisms of the F -space $L_0(= L_0(0, 1))$ of all real (or complex) measurable functions on $(0, 1)$ with the topology of convergence in measure; this followed an earlier paper of Berg, Peck and Porta [1] on the complemented subspaces of L_0 . We first state Kwapien's theorem for future reference. The version given below is a slight modification of Kwapien's original statement.

Theorem 1.1. (Kwapien) *Let K be a Polish space and let λ be a probability measure defined on the Borel subsets \mathcal{B} of K . Let (X, Σ, μ) be a measure space with $\mu(X) = 1$. Let $T : L_0(K, \mathcal{B}, \lambda) \rightarrow L_0(X, \Sigma, \mu)$ be a continuous linear operator; then T takes the form*

$$(1.1.1) \quad Tf(x) = \sum_{n=1}^{\infty} a_n(x)f(\sigma_n x) \quad \mu\text{-a.e.} \quad f \in L_0(K)$$

where

(i) $\{a_n\}$ is a sequence of elements of $L_0(X, \Sigma, \mu)$ such that $\mu\{x : a_n(x) \neq 0 \text{ for infinitely many } n\} = 0$.

(ii) $\sigma_n : X \rightarrow K$ is a sequence of $\Sigma - \mathcal{B}$ measurable mappings satisfying the condition that if $\lambda(A) = 0$ then $\mu(\sigma_n^{-1}(A) \cap \{x : a_n(x) \neq 0\}) = 0$.

Conversely if $\{a_n\}$ and $\{\sigma_n\}$ satisfy (i) and (ii), then (1.1.1) defines an operator from $L_0(K)$ into $L_0(X)$.

We have modified Kwapien's statement to the extent that we assume each σ_n Σ -measurable and not merely measurable with respect to the completion of Σ . This is permissible since K is Polish.

Let us now state Theorem 1.1 in a different form.

Theorem 1.2. *Let K be a compact metric space and let λ be a probability measure on K . Let (X, Σ, μ) be a measure space with $\mu(X) = 1$. Let $T : L_0(K, \mathcal{B}, \lambda) \rightarrow L_0(X, \Sigma, \mu)$ be a continuous linear operator; then T takes the form*

$$(1.2.1) \quad Tf(x) = \int_K f(t)dv_x(t) \quad \mu\text{-a.e.}, \quad f \in L_0(K)$$