

# *On the Duals of Some Spaces of Locally Schlicht Functions*

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**1. Introduction.** This paper studies some properties of Banach spaces of analytic functions. In particular a considerable portion of the work in this paper concerns a space  $X$  of locally univalent functions on the unit disk. The functions of  $X$  are suitably normalized and are required to satisfy the condition

$$(1 - |z|) \left| \frac{f''(z)}{f'(z)} \right| \leq M < \infty$$

for  $|z| < 1$ . In [12] Ch. Pommerenke has analyzed the function theoretic properties of the functions in  $X$ . There is a linear space structure imposed on the set  $X$ . These operations were first introduced by H. Hornich in [9]. Namely if  $f$  and  $g$  are in  $X$  and  $\alpha \in \mathbb{C}$  we define the vector sum and scalar multiplication by

$$f \oplus g(z) \equiv \int_0^z f'(t)g'(t)dt$$

$$\alpha \otimes f(z) \equiv \int_0^z (f'(t))^\alpha dt.$$

The inequality above allows us to equip  $X$  with a natural norm and with this norm  $X$  is a Banach space. We establish theorems which show that  $X$  is continuously isomorphic to certain maximal subspaces of the Bloch space and to a maximal subspace of a space of holomorphic functions introduced by Flett [7].

Further, we define a natural operator  $j$  mapping  $X$  into another Banach space of holomorphic functions  $EO_0$ . The latter space was introduced by Rubel and Shields in [13] and several properties of the functions in the space were discussed there. They obtained useful duality relationships in that same paper. We prove that  $j$  is a completely continuous operator and that  $Y = jX$  is a dense subspace of its range. The operator  $j_0$  defined by considering  $j$  as an operator in  $X$  onto  $Y$  has adjoint  $j_0^*$  and  $j_0^*Y^*$  is a weak-star dense linear subspace of  $X^*$ .

In the original version of this paper we observed that  $j_0^*$  was not an onto mapping but asked the question as to whether or not the range of  $j_0^*$  was norm dense in  $X^*$ . The referee of this paper has supplied a proof that shows that the