

The Spectra of a Class of Operators on the Disc Algebra

HERBERT KAMOWITZ

Introduction. The class of operators in the title consists of operators which are products of an automorphism followed by a multiplier. Specifically, if A is the disc algebra, the sup-norm algebra of functions analytic on the open unit disc D and continuous on \bar{D} , then the operators T we consider have the form $Tf(z) = u(z)f(\varphi(z))$ where $f \in A$, $u \in A$, $z \in \bar{D}$ and φ is a linear fractional transformation mapping D onto itself.

Two special cases should be noted. If $\varphi = z$, then T is simply a multiplier and it is well known that the spectrum of T is equal to the range of u . On the other hand, if $u \equiv 1$, then T is an automorphism of A and either the spectrum of T is a finite union of finite subgroups of the circle or else the spectrum of T equals the unit circle. The spectra of automorphisms of Banach algebras have been considered in more generality in [4], [5], [8].

Throughout this note, $\sigma(T)$ denotes the spectrum of T , φ_n denotes the n^{th} iterate of φ , φ_{-1} is the inverse of φ and $\varphi_{-k} = (\varphi_{-1})_k$, $D = \{z : |z| < 1\}$ and $\Gamma = \{z : |z| = 1\}$. A point $b \in \bar{D}$ is a *fixed point* of φ if $\varphi(b) = b$, and the spectral radius of T is denoted by $\|T\|_{\text{sp}}$ which equals

$$\lim_{n \rightarrow \infty} \|T^n\|^{1/n}.$$

Linear fractional transformations φ mapping D onto D are characterized as *parabolic*, *hyperbolic* or *elliptic* according as φ has exactly one fixed point on Γ , two fixed points on Γ , or one fixed point in D , respectively.

The main results about the spectra of the operators $T : Tf(z) = u(z)f(\varphi(z))$ will show that the spectra depend largely on the behavior of u at the fixed points of φ . Specifically, in this note we prove the following four theorems.

Theorem. *Suppose $u \in A$ and φ is a linear fractional transformation mapping D onto D . Let*

$$Tf(z) = u(z)f(\varphi(z)) \quad \text{for } f \in A, z \in \bar{D}$$