

Internal Constraints in Boundary Value Problems of Continuum Mechanics

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“Rigidity is an ideal which a body approaches without attaining.”

B. Russell

The Analysis of Matter

Introduction. Internally constrained materials in continuum mechanics serve to describe idealized material models for which certain classes of deformations are ruled out *a priori*. The applicability of such models for describing the behavior of real materials can be questioned however, considering the fact that no real material is constrained in the precise sense of the term. Incompressibility, for example, more familiar perhaps in connection with fluid mechanics, is an internal constraint which restricts the motion of the continuum to volume preserving deformations. Although no real material is really incompressible, for certain materials, *e.g.* water, oils, rubber, the effort required to produce a change in volume is orders of magnitude higher than the effort needed for producing, say, a shearing deformation. Often, then, in classical fluid mechanics and the theory of rubber elasticity, the compressibility effects are neglected altogether, in order to take advantage of the simplifying features of the constrained theory. Other types of internal constraints are also employed in practice for the same reasons. For example, the constraint of inextensibility is used in connection with fiber-reinforced materials and ultimately, in certain circumstances bodies are conveniently assumed to be rigid (another internal constraint).

The passage to the internally constrained materials is as a rule accompanied with drastic changes in the formulation of the corresponding boundary value problems. (Compare, for example, the partial differential equations of motion of compressible and incompressible fluids in Part III.) It is therefore by no means obvious *a priori*, that the solutions obtained from the reduced version of the equations are in any sense close to the solutions of the original problem. The practice of employing internally constrained models for almost constrained materials thus should be justified.

In Parts I and II to follow, we study two boundary value problems for elastostatics of nonlinear and linear hyperelastic materials. The effects of slight