

# *The Stefan Problem for a Pseudo-Heat Equation*

WILLIAM RUNDELL

**Introduction.** A classical problem for the heat equation is the so-called one phase Stefan problem. In the one-dimensional case this can be stated as follows. Consider a thin solid body occupying the half-line  $x \geq 0$ , and suppose that the temperature of the body is uniformly at its melting point. At  $x = 0$  the temperature is kept at  $f(t)$  degrees for all times  $t \geq 0$  where  $f(t)$  is assumed to be greater than or equal to the melting point of the solid. The solid then begins to melt, and for each time  $t$  the liquid will occupy an interval  $0 < x < s(t)$  while the solid occupies the region  $x > s(t)$ . The problem is to find the temperature at each point in the water for  $t > 0$  and to find the position of the phase or free boundary  $x = s(t)$  separating the solid and liquid.

A great deal is known about the Stefan problem for the heat equation (or indeed for parabolic equations) in that the major questions of existence, uniqueness, smoothness and shape of the free boundary have by and large been answered [5], [6].

In many physical models, if one imposes certain higher order corrections, then instead of the heat equation one has an equation of pseudoparabolic or Sobolev-Galpern type. For example in [2] Chen and Gurtin consider a two-temperature theory of thermodynamics in which the constitutive assumptions are that the energy, entropy, heat flux and thermodynamic temperature depend on the conductive temperature and its first two spatial gradients. The conductive temperature  $\theta$  can be shown to satisfy an equation of pseudoparabolic type

$$(0.1) \quad \epsilon \Delta \theta_t - \theta_t + \frac{k}{c} \Delta \theta = 0$$

where  $c$ ,  $k$ ,  $\epsilon$  are positive constants,  $c$  being the specific heat and  $k$  the conductivity. In practice the  $\epsilon$  that appears in the correction term  $\delta \Delta \theta_t$  is generally small in relation to the other physical constants in the system.

For further information regarding equations of Sobolev type one is referred to the survey in Chapter 3 of the book by Carroll and Showalter [1].

In this paper we propose to consider the problem of the melting of a solid with the heat equation replaced by an equation of pseudo-parabolic type. We