

Diffeomorphism Groups of Cobordant Manifolds

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1. Introduction. In this paper, we develop a method for the study of $\pi_0(\text{Diff } M^n) (D(M))$, the group of isotopy classes of orientation preserving diffeomorphisms of an oriented smooth manifold M^n (the group of pseudo-isotopy classes of orientation preserving diffeomorphisms of M^n).

It seems that the known techniques in differential topology are not effective in studying these groups except for a few specific manifolds; S^n by Kervair and Milnor [10], $S^p \times S^q$ by Browder [4], Levine [12], Sato [16] and Terner [19], T^n by Hsiang and Sharpe [9] and D^n by Cerf [6].

Our method is very elementary and it can be used for any bounding manifold, but there are many difficulties in carrying out the program.

In [12], Levine showed that to calculate $D(S^p \times S^q)$, it is enough to study the framed imbeddings of standard spheres into standard spheres.

In the next three sections of this paper, we show that almost the same statement is true for any closed oriented manifold that bounds. That is, for such an M^n , there exists a sequence of manifolds, $\{M_i\}_{i=1}^{\ell}$, such that $D(M^n)$ (or $\pi_0(\text{Diff } M^n)$ with some restrictions on M^n) can be calculated by studying the framed imbeddings of standard spheres into M_i , $i = 1, 2, \dots, \ell$.

In Section 5, we give an application and show the following theorem. Let E_c be the total space of an oriented $(p + 1) -$ disk bundle over S^q with characteristic map $c: S^{q-1} \rightarrow SO_{p+1}$. We regard $\partial E_c = S^p \times D_+^q \cup_c S^p \times D_-^q$. Let α be the inclusion of $S^p \times D_+^q$ into ∂E_c and $\bar{\alpha} = \alpha|_{S^p \times \{0\}}$. Define $\pi_0(\text{Diff } \partial E_c, \bar{\alpha})$ to be the subgroup of $\pi_0(\text{Diff } \partial E_c)$ of the element that has a representative fixing the points in $\text{Im}(\bar{\alpha})$. Notice that $\pi_0(\text{Diff } \partial E_c, \bar{\alpha})$ has at most index 2 if $2 \leq p < q$.

Theorem 5.7. $\pi_0(\text{Diff } \partial E_c, \bar{\alpha})$ satisfies the following exact sequences.