

On Multipliers of H^p Spaces

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1. Introduction. Let H^p , $1 \leq p < \infty$, denote the Banach space of functions $f(z) = \sum_0^\infty \hat{f}(n)z^n$ analytic in $\{z: |z| < 1\}$ for which

$$M_p(f, r) = \left(\frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^p d\theta \right)^{1/p}$$

is bounded as a function of r when $0 < r < 1$. Then $\lim_{r \rightarrow 1-} f(re^{i\theta}) = f^*(e^{i\theta})$ exists a.e. and

$$\|f\|_p = \|f^*\|_{L^p} = \lim_{r \rightarrow 1-} M_p(f, r)$$

is a complete norm on H^p . (See reference [1] for these and other facts about H^p spaces.)

Let

$$\Lambda(z) = \sum_0^\infty \lambda_n z^n \text{ in } \{z: |z| < 1\},$$

and write

$$f * \Lambda(z) = \sum_{n=0}^\infty \lambda_n \hat{f}(n) z^n.$$

The sequence $\lambda = (\lambda_n)$ is said to be an $H^p - H^q$ multiplier if whenever $f \in H^p$ then $f * \Lambda \in H^q$. It follows from the closed graph theorem that $T_\lambda f = f * \Lambda$ is a bounded linear operator from H^p into H^q and so there is a constant C_λ so that $\|T_\lambda f\|_q \leq C_\lambda \|f\|_p$. Denote the set of $H^p - H^q$ multipliers by (H^p, H^q) .

Hardy and Littlewood [3] have proved the following theorem.

Theorem A. If $1 \leq p \leq 2 \leq q$ and $p^{-1} - q^{-1} = 1 - \sigma^{-1}$ and if

$$(1.1) \quad M_\sigma(\Lambda', r) \leq C(1 - r)^{-1}$$

then $\lambda \in (H^p, H^q)$.

The condition (1.1) is sometimes also necessary. For let $\varphi_r(z) = (1 - r^2)(1 - rz)^{-2}$ when $0 < r < 1$. Since $|\varphi_r(e^{i\theta})| = P_r(\theta)$, the Poisson kernel, it follows that $\|\varphi_r\|_1 = 1$. Thus if $\lambda \in (H^1, H^q)$ then $\|T_\lambda \varphi_r\|_q \leq C_\lambda$ independently of