

# *Hyperinvariant Subspaces of $C_{11}$ Contractions, II*

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**§1. Introduction.** For a bounded linear operator  $T$  acting on a complex, separable Hilbert space, let  $\text{Lat } T$  and  $\text{Hyperlat } T$  denote its invariant subspace lattice and its hyperinvariant subspace lattice, respectively. In [6], we investigated  $\text{Hyperlat } T$  when  $T$  is a completely non-unitary  $C_{11}$  contraction with finite defect indices. We showed that for such contractions  $\text{Hyperlat } T$  is preserved, as a lattice, under quasi-similarity. We continue our investigation in this paper. For the contraction considered we give an explicit description of the elements in  $\text{Hyperlat } T$  in terms of the characteristic function and the functional model of  $T$  (Theorem 3.5). If  $T$  is also multiplicity-free, by combining with a result of Clark [1] we obtain a complete description of the elements in  $\text{Lat } T$  (Theorem 4.1). Our results generalize the corresponding results of Sickler [2] in that he only considered completely non-unitary contractions with outer scalar-valued characteristic functions, which are special cases of the operators we consider here. Indeed, the proof of Theorem 3.5 (comprising Lemmas 3.1–3.4) follows the general outline of Sickler's proof ([2], §4). As applications, Theorem 3.6 says that any element in  $\text{Hyperlat } T$  is the kernel and the range of some operators in  $\{T\}''$ , the double commutant of  $T$ , and Theorem 4.3 gives a necessary and sufficient condition that  $\text{Lat } T = \text{Hyperlat } T$ .

The determination of the elements in  $\text{Hyperlat } T$  for weak contractions will appear in a subsequent paper [7].

In §2 below, we briefly review the background needed for our work. §3 gives the complete description of the elements in  $\text{Hyperlat } T$  mentioned above. In §4, we restrict our considerations to multiplicity-free  $C_{11}$  contractions.

**§2. Preliminaries.** We follow the notation and the terminology used in [6].

Let  $\mathbb{C}$  be the complex plane and let  $L_n^2$  and  $H_n^2$  denote the standard Lebesgue and Hardy spaces of  $\mathbb{C}^n$ -valued functions defined on the unit circle  $C$ . We use  $t$  to denote the argument of a function defined on  $C$ . A statement involving  $t$  is said to be true if it holds for almost all  $t$  with respect to the Lebesgue measure.

Assume that  $T$  is a completely non-unitary (c.n.u.) contraction with defect indices  $d_T = d_{T^*} \equiv n < \infty$ . In the discussion of the following we shall consider