

# *Hypersurfaces with Bounded Levi Form*

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We consider an oriented real hypersurface  $S \subset \mathbb{C}^{n+1}$  defined by

$$S = \{z \in \mathbb{C}^{n+1} : r(z) = 0\}, \quad r \in C^2(\mathbb{C}^{n+1}),$$

$\nabla r \neq 0$  on  $S$ . Let  $TS$  denote the real tangent bundle of  $S$  and  $HS = TS \cap iTS$  denote the holomorphic or *CR* tangent bundle. If we set  $d^c = i(\bar{\partial} - \partial)$  and define

$$\Lambda = \frac{dd^c r}{|\nabla r|}$$

then the action of the 2-form  $\Lambda$  on vectors of  $HS$  is independent of the choice of  $r$ . (Changing the sign of  $r$  reverses both the sign of  $\Lambda$  and the orientation of  $S$ .) Following Hermann [5], we define the *Levi form* of  $S$  as  $\Lambda$ , viewed as a Hermitian form on  $HS \otimes \mathbb{C}$ . In certain applications related to potential theory (see [2] and [6]) this normalized Levi form arises quite naturally.

Since  $\Lambda$  is obtained from part of the second fundamental form of  $S$ , it is not unreasonable that it will have some properties similar to curvatures. The specific analogies between Levi form and curvature discussed here deal with bounding the radius of the domain over which  $S$  can be defined as a non-parametric surface. Bounds may be given in terms of various functions of the curvatures of  $S$  (see Aminov [1], Chern [3], and Flanders [4] and the references there).

Here we consider the situation where  $(w, z_1, \dots, z_n)$  are coordinates in  $\mathbb{C}^{n+1}$ , and  $S$  is the graph of a function  $\varphi(\operatorname{Re} w, z_1, \dots, z_n)$ . The radius of the set over which  $S$  may be defined is then bounded in terms of the Levi form. For certain special sets, another quantity,  $S_2(\Lambda)$  may be used to bound the radius.

**1. The Levi determinant in divergence form.** Let  $i : S \rightarrow \mathbb{C}^{n+1}$  denote inclusion, and let  $i^*dd^c r$  be the pullback of  $dd^c r$ . Then

$$dr \wedge d^c r \wedge dd^c r = dr \wedge d^c r \wedge i^*dd^c r,$$

and this (2,2)-form is seen to contain all the information of  $\wedge$ . We write

$$r_j = \frac{\partial r}{\partial z_j}, \quad r_{\bar{k}} = \frac{\partial r}{\partial \bar{z}_k}, \quad r_{j\bar{k}} = \frac{\partial^2 r}{\partial z_j \partial \bar{z}_k} \quad \text{and} \quad dV = \prod_{j=1}^n \frac{i}{2} dz_j \wedge d\bar{z}_j.$$