

Elliptic Perturbation of a First Order Operator with a Singular Point of Attracting Type

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§1. Introduction. Let D be a compact domain in R^n with smooth boundary Γ . We shall discuss the asymptotic behavior as $\epsilon \rightarrow 0$ of the solutions of the first boundary-value problem

$$(1.1) \quad L_\epsilon u = \epsilon \left(\sum_{i,j=1}^m a_{ij}(x) u_{x_i x_j} + \sum_{i=1}^m a_i(x) u_{x_i} \right) + \sum_{i=1}^m b_i(x) u_{x_i} = 0 \text{ in } D$$

$$(1.2) \quad u(x) = \varphi(x), \quad x \in \Gamma.$$

Here $x = (x_1, x_2, \dots, x_m) \in R^m$ and we suppose that

$$\nu^{-1} \sum_{i=1}^m \xi_i^2 \leq \sum_{i,j=1}^m a_{ij} \xi_i \xi_j \leq \nu \sum_{i=1}^m \xi_i^2, \quad a_{ij}(0) = \delta_{ij}^i.$$

This problem arises in the study of the effect of small random perturbations on dynamical systems (see [11], [7], [8]). It is known that the behavior of the characteristics of the reduced equation

$$(1.3) \quad \sum_{i=1}^m b_i u_{x_i} = 0$$

i.e. the integral curves of the system

$$(1.4) \quad \frac{dx_1}{b_1(x)} = \frac{dx_2}{b_2(x)} = \dots = \frac{dx_m}{b_m(x)},$$

is of decisive importance in this connection.

Let the positive direction along the characteristics coincide with the direction of the vector $b = (b_1, \dots, b_m)$, and $u_\epsilon(x)$ be a solution of the problem (1.1), (1.2). Suppose the characteristic curve which starts at a point $P \in D$ leaves the domain D at $P_0 \in \Gamma$. In that case, using classical methods (see *e.g.* [1], [5], [12]) it can be shown, that $u_\epsilon(P) \rightarrow \varphi(P_0)$. In this paper we consider the case that the system (1.4) has a single singular point which is of attracting type and all characteristics that start at the boundary reach the singular point (turning point).