

Weighted Interpolation in Banach Function Spaces

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§1. Introduction. A. P. Calderón [7] has shown that a linear operator T is simultaneously of weak type (p_0, q_0) and (p_1, p_1) , $1 \leq p_0 < p_1 < \infty$, $1 \leq q_0$, $q_1 \leq \infty$, if and only if for each measurable function f

$$(1.1) \quad (Tf)^*(t) \leq A \left\{ t^{-1/q_0} \int_0^{tm} s^{1/p_0} f^*(s) \frac{ds}{s} + t^{-1/q_1} \int_{tm}^{\infty} s^{1/p_1} f^*(s) \frac{ds}{s} \right\},$$

where A is a constant, g^* the decreasing rearrangement of g and m the slope of the line passing through the points $(1/p_0, 1/q_0)$, $(1/p_1, 1/q_1)$. That (1.1) may serve as definition of weak type operators also for the case that one of the indices is not finite was observed by Bennett [1] and Sharpley [13]. Using this fact and Hardy's inequality one obtains strong mapping properties of T , via the Marcinkiewicz theorem, its extension to Orlicz spaces and spaces involving weights. (See for example [2], [3], [9], [10], [11] or [15].)

Let X_0, X_1 be two Banach spaces contained in a linear Hausdorff space \mathfrak{X} , such that the identity mapping of X_i , $i = 0, 1$ in \mathfrak{X} is continuous. Denote the elements of X_i by f_i with norm $\|f_i\|_i$ and let $X_0 + X_1$ be the space of functions f , such that $f = f_0 + f_1$. The functional

$$(1.2) \quad K(t; f) \equiv K(t; f, X_0, X_1) = \inf_{f = f_0 + f_1} (\|f_0\|_0 + t\|f_1\|_1)$$

for $0 < t < \infty$, defines a function norm which is concave and hence $K(t; f)/t$ is decreasing. If L^p denotes the Lebesgue spaces of p^{th} power Lebesgue integrable functions, then it is well known ([4, Theorem 5.21]) that for $0 < p < \infty$

$$(1.3) \quad K(t; f, L^p, L^\infty) \sim \left\{ \int_0^{tp} f^*(s)^p ds \right\}^{1/p},$$

with equality when $p = 1$.

See [4], [8] and [16] for other computations of this functional. In general, if $f \in X_0 + X_1$, then there exists a nonnegative decreasing function $k(t; f)$, such that

$$k(t; f) = \int_0^t k(s; f) ds.$$