

Branching of Singularities for a Class of Hyperbolic Operators

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Introduction. In this paper, we are considering a class of operators of the form $p = p_1 p_2 + R$, where p_i is of order one with real principal symbol p_i ($i = 1, 2$) and R is of order zero. We assume that $\{p_1, p_2\} \neq 0$ on the manifold of double characteristic points $\Sigma = \{p_1 = p_2 = 0\}$, so that the integral curves of H_{p_1} and H_{p_2} start transversally from Σ .

For these operators, the propagation of singularities has been studied by Ivrii [4]; a parametrix is given in [1], along with the microlocal regularity of the solutions (see also [6] for some special cases), which depends on R .

The main theorem of the present paper gives precise results on how bicharacteristic curves, bearing singularities of a solution u of $Pu = 0$, “branch” across Σ , and on the microlocal regularity of u near each branch.

I. Notations; statement of the results. Let us denote the coordinates in \mathbb{R}^2 by (x, t) and let $P(t, D_x, D_t) = \partial_t^2 - t^2 \partial_x^2 + \pi(t, D_x)$ be a hyperbolic operator (where π is a classical pseudo-differential operator of order 1 in the x -variables, depending smoothly on $t \in \mathbb{R}$).

The Cauchy problem with data on $t = 0$ has been studied in [1] for a general class of hyperbolic operators which contains P . Let us recall the main theorem of [1] in the particular case of the operator P :

Theorem 1. *There are operators $E_{\pm} : \mathcal{E}'_{+1}(\mathbb{R}) \rightarrow \mathcal{D}'(\mathbb{R}^2)$ (where $\mathcal{E}'_{+1}(\mathbb{R})$ is the space of distributions $u \in \mathcal{E}'(\mathbb{R})$, $\text{WF}(u) \subset \{\xi > 0\}$) such that:*

- i) $PE_{\pm} = 0$, modulo C^{∞} .
- ii) *For any $t_0 > 0$, $E_{\pm}|_{t > t_0}$ is (modulo a smoothing operator depending on t_0) a Fourier-integral operator with canonical relation $C(p_{\pm})$ and order $m_{\pm} - (1/4)$, where*

$$m_{\pm} = -\frac{1}{4} \pm \frac{1}{4} (\text{Im } \pi_1(0, +1)).$$

Here, $p_{\pm} = \tau \pm t\xi$, and $C(p_{\pm})$ is the canonical relation associated with the Cauchy problem for $D_t \pm tD_x$.