

Stable p -Equivalences of Stunted Complex Projective Spaces

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1. Introduction. Let p be a prime number. Spaces X and Y are stably p -equivalent if there exist integers r and s and a p -equivalence $f: \Sigma^r X \rightarrow \Sigma^s Y$, i.e. f induces an isomorphism in Z_p -cohomology. Let $CP_n^{n+k} = CP^{n+k}/CP^{n-1}$ denote the stunted complex projective space. Let $\nu_p(m)$ be the highest power of p that divides m .

Our main results are the following two theorems.

Theorem 1.1. i) CP_n^{n+2} and CP_m^{m+2} are stably 2-equivalent if and only if $n \equiv m(8)$ or $n \equiv m \equiv 2, 3(4)$.

ii) CP_n^{n+3} and CP_m^{m+3} are stably 2-equivalent if and only if $n \equiv m(8)$ or $n \equiv m \equiv 2(4)$.

iii) CP_{8k}^{8k+4} and $CP_{8\ell}^{8\ell+4}$ are stably 2-equivalent if and only if $k \equiv \ell(8)$ or $\nu_2(k) = \nu_2(\ell)$. Dually, $CP_{8k}^{8k-\frac{1}{5}}$ and $CP_{8\ell}^{8\ell-\frac{1}{5}}$ are stably 2-equivalent if and only if $k \equiv \ell(8)$ or $\nu_2(k) = \nu_2(\ell)$.

iv) If CP_n^{n+4} and CP_m^{m+4} are stably 2-equivalent, then

a) $n \equiv m(8)$

b) if $n \equiv 1$ or $2(8)$, then $n \equiv m(16)$

c) if $n \equiv 1$ or $10(16)$, then $n \equiv m(32)$.

Theorem 1.2. Let p be an odd prime.

i) If $i < p - 1$, then CP_n^{n+i} and CP_m^{m+i} are stably p -equivalent.

ii) If $0 \leq i < p - 1$, then $CP_n^{n+p-1+i}$ and $CP_m^{m+p-1+i}$ are stably p -equivalent if and only if $n \equiv m(p)$ or $1 \leq n, m \leq p - 1 - i(p)$.

iii) If $0 \leq i < p - 1$, then $CP_n^{n+2(p-1)+i}$ and $CP_m^{m+2(p-1)+i}$ are stably p -equivalent if and only if (a) $n \equiv m(p^2)$ or (b) $n \equiv m(p)$ and $1 \leq n, m, n + 2p - 1, m + 2p - 1 \leq p^2 - 1 - i(p^2)$.

In [7] and [8] Feder and Gitler determine which stunted complex projective spaces have the same stable homotopy type. Our results indicate that there are more p -equivalences than one might expect from naively localizing their result. Indeed, our theorems give the following examples of pairs $([X], [Y])$ of distinct stable homotopy types which are stably p -equivalent for all primes p :