

Continuity of the Temperature in the Stefan Problem

LUIS A. CAFFARELLI & AVNER FRIEDMAN

Introduction. In this paper we prove the following theorem:

(0.1). The temperature in the one phase n -dimensional Stefan problem is continuous.

This result is well known if $n = 1$ [8]. For $n = 2$ it was recently proved by Caffarelli [4] as a by-product of a study of the free boundary. The present proof, which is valid for all n , does not require any preliminary analysis of the free boundary.

In Section 1 we state the assertion (0.1) more precisely, giving also a modulus of continuity for the temperature.

In Section 2 we prove the continuity of the temperature at $t = 0$, and also show that the set occupied by water is strictly increasing at $t = 0$.

In Section 3 we prove the continuity of the temperature for $t > 0$ and establish a modulus of continuity, provided $n \geq 3$. The case $n = 2$ is considered in Section 4, and a better modulus of continuity is obtained in this case.

In Section 5 we derive some estimates that allow us, in particular, to (i) establish the continuity of the temperature under somewhat different conditions from those in Section 1, and (ii) obtain a uniform modulus of continuity on the temperature, down to $t = 0$.

Finally, in Section 6 we give some applications of the continuity results to the study of the free boundary.

1. The main results. For the sake of definiteness we consider here the exterior Stefan problem, that is, the water is situated outside a fixed set.

Let G be a bounded domain in R^n whose boundary ∂G consists of two connected manifolds, Γ and Γ_0 , such that Γ_0 lies in the interior of Γ . Denote the interior of Γ_0 by G_0 , and let $G_1 = G \cup \bar{G}_0$.

In the classical formulation of the Stefan problem one seeks a pair of functions, the temperature $\theta(x, t)$ and the water-ice interface $t = s(x)$ such that