

Parametrixes and Propagation of Singularities for Operators with Non-involutive Characteristics

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0. Introduction. In this paper we study pseudodifferential operators P with real principal symbol. It is assumed that the characteristic variety of P is the union of two smooth hypersurfaces with non-involutive intersection. The construction of microlocal parametrixes enables us to study the propagation of singularities. This is a natural continuation of work begun by the author in [6], [7].

Working independently, Ivrii [10], [11] and Melrose [12], [13] have obtained essentially the same results as we have. The notes of Ivrii are quite brief and contain no proofs, while the manuscript of Melrose contains a proof somewhat different from ours. Also see Alinhac [0], for the study of a closely related system, and Kashiwara, Kawai and Oshima [16], [17] (or Miwa [18]) for a more general discussion in the real analytic case.

We believe that the explicit formulae of Sections 3 and 4 provide a short, direct approach that will be useful in other multiple characteristic problems.

In Section 1 we state our results and discuss some examples. In Section 2 we reduce P to the form $L = t\partial_t - B_0(x, D_x)$, $B_0 \in L_c^0(\mathbb{R}^n)$. In Section 3, we discuss parametrixes and propagation of singularities for L under no condition on the lower order terms. In Section 4 we discuss these problems under certain assumptions on the subprincipal symbol.

1. Statements of results. Notation used and not defined here is standard. See Hörmander [8] and Duistermaat-Hörmander [4].

Let $X \subset \mathbb{R}^{n+1}$ be an open set. By $L_c^m(X)$ we mean the collection of all $A \in L_{1,0}^m(X)$ whose full symbol has an asymptotic expansion in positively homogeneous terms of orders $m, m-1, m-2, \dots$. Let $P_j \in L_c^{m_j}(X)$ for $j = 1, 2$ and $Q \in L_c^{m_1+m_2-1}(X)$, with principal symbols p_j and q . Assume the p_j are real-valued.

We will be studying pseudodifferential operators

$$(1.1) \quad P = P_1 P_2 + Q$$