

Framed Manifold Models for the Elements β_1

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0. Introduction. In this paper we assume the ideas and notation of [18]. Therein, machinery was developed suitable for detecting framed manifolds representing the well-known stable homotopy elements $\beta_1 \in {}_p\mathcal{S}_{2p^2 - 2p - 2}$ (see [22]) for each odd prime p (recall that in this dimension the p -component of the stable stem is \mathbb{Z}_p , and β_1 is a generator). We here apply that machinery, and so display one possible construction of the β_1 's.

We note in passing that β_1 , as originally constructed, lies in the p -fold Toda bracket $\langle \alpha, \alpha, \dots, \alpha \rangle$, where α generates ${}_p\mathcal{S}_{2p-3} \cong \mathbb{Z}_p$ and is realised on the sphere S^{2p-3} . Thus using the framed bordism Massey product methods of [2], [4] we already have some model for β_1 . However, this contains a great deal of (geometrical) indeterminacy; it is our purpose to avoid this problem and present a precise model.

The main calculations occur in §3, §4, §5 and are fairly short. The first two sections are not central, and somewhat longer. Notwithstanding, we feel they discuss some interesting aspects of the problem.

In §1 we briefly recall what we need from [18], and give a condition for a framed boundary to be reframable to β_1 .

In §2 we define the manifolds $N^{2p^2 - 2p - 2}$ (usually abbreviated to N) as extended powers, and describe the origins of a bounding framing. We also exhibit some simple (co)homology properties of N ; these are augmented in §3 by some discussion of the action of the Steenrod algebra, abetted by a particularly useful reframing.

We compute the adjoint fundamental class, modulo irrelevancies, in §4. After making an observation on the Dyer-Lashof algebra in §5, we can then prove our main theorem (5.3).

Throughout, $H^*()$ and $H_*()$ denote (co)homology with \mathbb{Z}_p coefficients, with p a fixed odd prime. g_n will always denote a generator of $H_n(S^n)$. It is convenient also to reserve the letter m for the quantity $(1/2)(p - 1)$.

Our work requires a considerable acquaintance with the mod p Dyer-Lashof algebra and operations Q^i , and some of the subtler aspects concerning composition products. These are admirably documented in the encyclopaedic [12, 13], to which we shall make detailed reference wherever necessary.