

# *Some Applications of Invariance for Parabolic Systems*

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**0. Introduction.** This paper is motivated by the following problem posed by R. Martin [6]. Given the two-dimensional parabolic system

$$\begin{aligned} a\Delta u - u_t &= -uv^\gamma, & a, b > 0 \\ b\Delta v - v_t &= uv^\gamma, & \gamma > 0 \end{aligned}$$

with nonnegative initial-boundary data of the Dirichlet or Neumann type, do nonnegative solutions exist globally?

We show that nonnegative solutions exist globally (Theorem 5) and that the zero solution is globally stable from above for the first boundary value problem with zero lateral boundary data (Theorem 7).

These results for this very special system are consequences of a general invariance result for semi-linear parabolic systems of the form

$$\begin{aligned} Lu &= Eu - u_t = f(x, t, u, \nabla u) \\ Bu &= 0, \end{aligned}$$

where  $E$  is a system of elliptic operators, and  $B$  is a system of mixed initial-boundary operators which are precisely defined in Section 1.

Roughly speaking, the idea of invariance is to construct a compact convex region  $\mathbb{S}$  in the range space which has the property that the “vector field”  $f$  points outward at each point of the boundary of  $\mathbb{S}$ . This condition essentially forces the global existence of a solution which remains in  $\mathbb{S}$  for all  $t > 0$ . Several authors have recently established results of this type ([1], [2], [9]). This is precisely stated as our Theorem 1.

This invariance result can be also obtained for certain space-time varying sets  $\Lambda$  which have compact convex cross section (Theorem 3). Such more general invariant sets are indeed needed to study the initially posed problem.

**1. Notation, definitions, and a key result.** Let  $\mathbb{R}^n$  denote  $n$ -dimensional real Euclidean space and let  $\Omega$  be a bounded domain in  $\mathbb{R}^n$  whose boundary  $\partial\Omega$  is an  $(n - 1)$ -dimensional manifold of class  $C^{2+\alpha}$ ,  $\alpha \in (0, 1)$ . Let  $\pi = \Omega \times (0, T)$  and  $\Gamma = (\partial\Omega \times [0, T]) \cup (\Omega \times \{0\})$  where  $0 < T \leq \infty$ .