

Analytic Moduli of Complex Vector Bundles

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Introduction. Let X be a compact connected complex manifold. For any integer $r \geq 1$, we denote by $H^1(X, GL(r, \mathcal{O}))$ the “cohomology set” parameterizing isomorphism classes of rank r vector bundles on X . This cohomology set may not naturally form an analytic space, essentially because of problems of topological nonseparation. Nevertheless, it may be hoped that, by excluding some atypical bundles, one is left with a family of vector bundles which is naturally parameterized by an analytic space. In recent years, considerable progress has been made in this study by algebraic geometers. In 1962, D. Mumford [16] specified a class of vector bundles (the “stable” bundles) on a Riemann surface X . The stable bundles naturally form an analytic variety (in fact, a smooth, quasiprojective algebraic variety); and stable bundles are generic, in the sense that, in any analytic family of vector bundles on X , the subfamily of stable bundles is the complement of an analytic subvariety. Recently Takemoto [21] and Gieseker [3] have provided two different definitions of stability, generalizing Mumford’s definition to the case where X is a projective manifold of arbitrary dimension. (Gieseker’s definition applies more generally to torsion-free sheaves on X , not just locally-free sheaves.) Maruyama [14] has shown that Gieseker’s definition generally yields an algebraic scheme.

In this paper we approach the above problem from an analytic point of view. The cohomology set $H^1(X, GL(r, \mathcal{O}))$ is regarded as the set of orbits for an action of a Banach-Lie group on a Banach-analytic space. Although the topology on this orbit space is generally highly non-separated, there is a generic locally Hausdorff subset, the *simple* bundles. We prove that the isomorphism classes of simple bundles are parameterized by a (not necessarily Hausdorff) analytic space S , and that this parameterization is natural, in the sense that S provides a universal deformation space for each of its points. Our arguments are an extension of ideas presented by R. C. Gunning in [8]. We have used the Banach-analytic tools of Douady [1] in order to provide S with its natural non-reduced structure sheaf.

In order to obtain a globally separated moduli space for bundles on X , some points of S must evidently be discarded. If X is assumed to be projective, the Hilbert polynomial provides an excellent means of determining which points to