

# *Residuality of Ergodic Measurable Transformations and of Ergodic Transformations which Preserve an Infinite Measure*

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**§0. Introduction.** Let  $(X, \mathbb{M}, \lambda)$  be a finite or  $\sigma$ -finite measure space. For the group  $\mathcal{M}$  of invertible, measure preserving transformations, or more generally for the group  $\mathcal{G}$  of invertible, measurable, non-singular transformations, it is natural to ask whether the subset of those transformations which are ergodic, is, in some sense, large. It was shown by Halmos [5] in 1944, that if  $(X, \mathbb{M}, \lambda)$  is a Lebesgue probability space (*i.e.* is isomorphic to the unit interval with Lebesgue measure), and the group  $\mathcal{M}$  is endowed with what he called the weak topology (we call this the coarse topology), which makes it a Baire space, the set of ergodic transformations is a residual, dense  $\mathcal{G}_\delta$  set. In this paper we prove an analogous result for the group  $\mathcal{G}$ , over a Lebesgue probability space, endowed with the coarse topology (as introduced by A. Ionescu Tulcea [10]). Using this we prove the analogous result for the group  $\mathcal{M}$  when  $(X, \mathbb{M}, \lambda)$  is a Lebesgue space of *infinite* measure (*i.e.* is isomorphic to the line with Lebesgue measure); this result has been proved in 1971, by U. Sachdeva [13]. For this last group we also prove that the conjugates of any fixed antiperiodic transformation are dense. We next consider various possible definitions of the coarse topology on  $\mathcal{G}$ , (using isometries in  $L^p$ ,  $1 \leq p < \infty$ , and varying  $p$ ) and show that they all yield the same topology. Finally we raise the possibility of proving similar results on measure preserving homeomorphisms of  $\mathbb{R}^n$  ( $n \geq 2$ ), analogous to those proved in 1941 by Oxtoby and Ulam [12] for  $[0, 1]^n$ .

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**§1. The coarse topology for measurable transformations.** Let  $(X, \mathbb{M}, \mu)$  be a Lebesgue space with  $\mu(X) = 1$ ,  $\mathcal{G} = \mathcal{G}(\mu)$  the group of measurable, invertible, non-singular transformations on  $(X, \mathbb{M}, \mu)$ , and  $\mathcal{M}(\mu)$  the subgroup consisting