

Scattering Theory for Hamiltonians Periodic in Time

JAMES S. HOWLAND

Introduction. This paper concerns the scattering theory of the Schroedinger evolution equation

$$(1) \quad i \frac{d\psi}{dt} = H(t)\psi \quad -\infty < t < \infty$$

where $H(t)$ is a self-adjoint Hamiltonian operator on a Hilbert space \mathcal{H} . It is desired to compare the asymptotic behavior of the solution of (1) with the solution $\psi_0(t)$ of a similar equation with Hamiltonian $H_0(t)$. If $U(t, s)$ and $U_0(t, s)$ are the corresponding unitary propagators for these equations, that is, the operators which give the solutions of the equations according to the formula

$$\psi(t) = U(t, s)\psi(s)$$

then the question reduces to the study of the wave operators

$$(2) \quad W_{\pm}(s) = s\text{-}\lim_{t \rightarrow \pm\infty} U_0(s, t)U(t, s)P$$

where P is some projection whose introduction may be necessary in order that the limit exist. The first things to be considered are usually existence of the limits, and the ranges of $W_{\pm}(s)$ and its adjoint. A substantial literature is devoted to the case where $H(t)$ is independent of t .

In a previous paper [9], the author considered the case where the difference $H(t) - H_0(t)$ goes to zero suitably as $t \rightarrow \pm\infty$. The method employed was suggested by a procedure of Classical Mechanics for reducing t to a spatial variable. One considers the operator

$$K = -i \frac{d}{dt} + H(t)$$

on the space $\mathcal{H} = L_2(-\infty, \infty; \mathcal{H})$, and the similar operator K_0 corresponding to $H_0(t)$. The study of $W_{\pm}(s)$ was shown to be essentially equivalent to the study of

$$\Omega_{\pm}(K_0, K) = s\text{-}\lim_{\sigma \rightarrow \pm\infty} e^{i\sigma K_0} e^{-i\sigma K}.$$