

More on Weak Estimates for Oscillating Kernels

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Introduction. In earlier papers [5], [11], we discussed the mapping properties of the kernels $k(x)e^{if(x)}$ with fairly general conditions on $f(x)$ and $k(x)$. Other authors such as Fefferman [6], [7], Carleson-Sjölin [3], Sjölin [12], [13] and Hörmander [9], have studied some of these kernels, while Zafran [15] solved an endpoint problem for ℓ^p into ℓ^p .

In this paper, using a fairly flexible technique, we solve an endpoint problem for $L^p(\mathbb{R})$ into $L^p(\mathbb{R})$. Here, we use an example "similar" to Zafran, namely

$$K_p(x) = |x|^{-1/p}\phi(x), K_1(x) = (1 + |x|)^{-1}\phi(x)$$

where

$$\phi(x) = e^{ix(\log|x|)^\eta}((\log(2 + |x|))^{-\delta},$$

$0 < \eta < 1$, $\delta = \frac{1}{2}(1 - \eta)$ and $1 < p < 2$, [see Theorem 2 and Theorem 3]. The case $p = 1$, which we do here, as far as we know has not been done before.

After this paper was written, we discovered that Theorem 1 could be used to solve an endpoint problem for the kernels

$$\frac{e^{i|x|^a}}{|x|^b}, a > 0, b < 1, a \neq 1, \frac{a}{2} + b - 1 > 0.$$

W. B. Jurkat and I are presently writing a paper that will give the precise L^p to L^q ($1 < p \leq q < \infty$) mapping properties for many kernels of the form $k(x)e^{if(x)}$.

§0. Notation and definitions. Now let us begin by defining

$$K_u(x) = \chi_u(x)K(x), \quad u > 0$$

where χ_u is the characteristic function of the interval $(-u, u)$ and we set

$$K^u(x) = \chi^u(x)K(x)$$

where $\chi^u(x) = 1 - \chi_u(x)$. Here, K stands for a locally integrable function, thus we write $K \in L_{loc}$.

By the symbol $(K)^\wedge$ we mean the Fourier transform of K .