

Maximum Principles for Degenerate Elliptic-Parabolic Operators

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This paper is concerned with maximum principles for degenerate elliptic-parabolic operators:

$$(1) \quad \sum_{i,j=1}^d a_{ij}(x) \frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i=1}^d b_i(x) \frac{\partial}{\partial x_i}.$$

As is well-known, there are two types of maximum principles, interior and boundary principles. Interior principles were established by Bony [1], Hill [2] and Redheffer [5] for operators of the divergence type:

$$(2) \quad \sum_{i=1}^d \left(\sum_{j=1}^d a_{ij}(x) \frac{\partial}{\partial x_j} \right)^2 + \sum_{i=1}^d b_i(x) \frac{\partial}{\partial x_i},$$

to which a general operator of the form (1) cannot always be reduced ([4]), and by Strook-Varadhan [6] for degenerate parabolic operators. Boundary principles were established by Hill [2] and Myers [3] for operators of type (2).

The object of this paper is to establish both types of maximum principles for degenerate elliptic-parabolic operators (1) by a unified treatment. Thus we state both principles in one single theorem in terms of the propagation set (Theorem 2). We, also, eliminate certain complicated statements in the latter principles (Remark 3).

1. Notation and results. Let L be a differential operator of the form (1) and let $X_0(x), X_1(x), \dots, X_d(x)$ be vector fields defined by

$$X_0(x) = \sum_{i=1}^d \left(b_i(x) - \sum_{j=1}^d \frac{\partial a_{ij}}{\partial x_j}(x) \right) \frac{\partial}{\partial x_i},$$

$$X_i(x) = \sum_{j=1}^d a_{ij}(x) \frac{\partial}{\partial x_j}, \quad 1 \leq i \leq d.$$

Here we assume that

$$(3) \quad \begin{aligned} a(x) &= (a_{ij}(x))_{d \times d} \in C^1(R^d, S_d), \\ b(x) &= (b_1(x), \dots, b_d(x)) \in C(R^d, R^d), \end{aligned}$$