

# *Measurability in a Banach Space, II*

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**Introduction.** This paper is intended as a continuation of [6], but much of it can be read independently. It deals with measurability in a Banach space; in particular with scalar measurability and Pettis integrability of functions with values in a Banach space. Most of the proofs are routine; it is hoped that the encyclopedic style in Section 3 and the examples in Section 5 will make the paper useful as a reference.

A brief summary of the paper follows. Section 1 reviews notation and terminology used in the paper. Section 2 contains a few results concerning weak Borel sets and weak Baire sets in a Banach space. Corollary 2.3 is a result of W. Schachermayer stating that if a Banach space  $X$  admits a Kadec norm (in particular if  $X$  is weakly compactly generated), then the canonical image of  $X$  in  $X^{**}$  is a weak\* Borel set. In general, the  $\sigma$ -algebra of weak Baire sets on a Banach space is not the same as the  $\sigma$ -algebra generated by the closed  $G_\delta$ -sets for the weak topology (Proposition 2.5).

In Section 3, several classes of Banach spaces more general than the weakly compactly generated spaces are discussed. (In fact, the classes considered are larger than the class of weakly  $K$ -analytic Banach spaces defined by M. Talagrand and the class of weakly countably determined spaces defined by L. Vařák.) The properties considered include: the weak topology is Lindelof; the weak topology is measure-compact; every bounded scalarly measurable function is Pettis integrable; the weak topology is realcompact; property (C), previously studied by R. Pol; the condition of Mazur; the weak\* topology on the dual is angelic.

Of the results proved in Section 3, only one will be mentioned here. The question of whether every measure-compact dual space has RNP (which would generalize the theorem [6, Proposition 1.8] that every Lindelof dual space has RNP) is equivalent to the question of whether there is a diffuse measure defined on all subsets of  $\mathbb{R}$ .

Section 4 is concerned with Pettis integrability of bounded scalarly measurable functions  $\varphi: \Omega \rightarrow X$  from a probability space  $(\Omega, \mathcal{F}, \mu)$  to a Banach space  $X$ . Some sufficient conditions for Pettis integrability are given: conditions on the measure space  $(\Omega, \mathcal{F}, \mu)$ , on the Banach space  $X$ , or both. If the weak topology on  $X$  is measure-compact (in particular if it is Lindelof), then every bounded scalarly measurable function with values in  $X$  is Pettis integrable