

# *Extremal Plurisubharmonic Functions for Certain Domains in $\mathbb{C}^2$*

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**1. Introduction.** We consider the following problem which is related to holomorphic mappings (see [1]) and capacity theory in  $\mathbb{C}^n$  (see [6] and [11]). Given smoothly bounded, strongly pseudoconvex domains  $\Omega_0 \subset \subset \Omega_1 \subset \subset \mathbb{C}^2$ , find  $u \in C^2(\bar{\Omega}_1 \setminus \Omega_0)$  such that

$u$  is plurisubharmonic on  $\Omega = \Omega_1 \setminus \bar{\Omega}_0$

$$(1) \quad \begin{aligned} \det \left( \frac{\partial^2 u}{\partial z_i \partial \bar{z}_j} \right) &= 0 \quad \text{on } \Omega \\ u &= 1 \quad \text{on } \partial\Omega_1 \\ u &= 0 \quad \text{on } \partial\Omega_0. \end{aligned}$$

Generalized solutions have been shown to exist (see [4] and [9]), and the generalized solution coincides with the function  $u(\zeta) = \sup v(\zeta)$ , where the supremum is taken over all  $v$  plurisubharmonic on  $\Omega$  such that  $\lim_{\zeta \rightarrow \zeta_1} v(\zeta) \leq 1$  for  $\zeta_1 \in \partial\Omega_1$  and  $\lim_{\zeta \rightarrow \zeta_0} v(\zeta) \leq 0$  for  $\zeta_0 \in \partial\Omega_0$ . For general domains, it was shown in [2] that there is no solution in  $C^2(\bar{\Omega}_1 \setminus \Omega_0)$ .

Here we show that the solution of (1) is smooth for certain domains which are small perturbations of the circular annulus  $A(R_0, R_1) = \{R_0^2 < |z|^2 + |w|^2 < R_1^2\}$ . We consider domains of the form

$$(2) \quad \begin{aligned} \Omega_j &= \{(z, w) : \log |w| < \psi^j(z), \quad |z| < R_j\}, \quad j = 0, 1 \\ \psi^j(z) &= \frac{1}{2} \log (R_j^2 - |z|^2) + \varphi^j(z) \end{aligned}$$

where  $\varphi^0, \varphi^1$  are small. Thus if  $\varphi^0 = \varphi^1 = 0$ , then  $\Omega_1 \setminus \bar{\Omega}_0 = A(R_0, R_1)$ . Because of the symmetry in the  $w$ -variable, we can introduce a free boundary problem with which (1) is reduced to a one-dimensional problem.

To motivate our construction, we describe the nature of the solution of (1) for a domain of the form (2); after the construction, it will be clear that these properties actually hold. If  $\partial\bar{\partial}u \neq 0$  then by the Frobenius theorem, the 2-form  $\partial\bar{\partial}u$  may be integrated to give a foliation  $\mathcal{M}$ , whose leaves are complex sub-