

On Holomorphic Curves and Meromorphic Maps in Projective Space

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Although there has been much work on the distribution of values of meromorphic functions of one complex variable and of meromorphic mappings of several complex variables, the subject of value distribution theory for holomorphic curves in $\mathbb{C}P^n$ ($n > 1$) is still in its infancy. In 1933, H. Cartan [3] showed that for a non-degenerate holomorphic curve in $\mathbb{C}P^n$, $\sum \delta(H_j) \leq n + 1$ where $\{H_j\}$ is any collection of hyperplanes in $\mathbb{C}P^n$ in general position. (Here δ denotes the Nevanlinna defect.) However it remains an open problem to determine a good upper bound for $\sum \delta(D_j)$ where $\{D_j\}$ is any collection of algebraic hypersurfaces of degree $p > 1$ in $\mathbb{C}P^n$ satisfying a suitable "general position" hypothesis. This paper represents an attempt to solve this problem and hopefully to encourage further work on this previously neglected area of value distribution theory.

Suppose $f: \mathbb{C} \rightarrow \mathbb{C}P^n$ is a holomorphic curve such that $f(\mathbb{C})$ is not contained in any algebraic hypersurface in $\mathbb{C}P^n$. Let $\{D_j\}$ be a collection of algebraic hypersurfaces of degree p in $\mathbb{C}P^n$. By considering the Veronese map on $\mathbb{C}P^n$ given by the monomials of degree p and applying Cartan's defect relations, it follows immediately that $\sum \delta(D_j) \leq \binom{n+p}{p}$ provided that the defining equations for the D_j are in "general position." However the upper bound $\binom{n+p}{p}$ is probably much too large. For instance, M. Green [5] proved the Picard theorem that f cannot omit $n + 2$ distinct irreducible algebraic hypersurfaces of arbitrary degree, suggesting that $n + 1$ may be an upper bound for the sum of the defects. Furthermore, Carlson and Griffiths [2] obtained a defect relation of the form $\sum \delta(D_j) \leq \frac{n+1}{p}$ for non-degenerate *equidimensional* maps $f: \mathbb{C}^n \rightarrow \mathbb{C}P^n$ provided that the D_j are smooth and have normal crossings. This suggests the corresponding result for holomorphic curves, but the methods of [2] do not easily carry over to holomorphic curves.

Our main result (Theorem 1) yields the defect relation $\sum \delta(D_j) \leq 2n$ for non-constant holomorphic curves $f: \mathbb{C} \rightarrow \mathbb{C}P^n$ given by polynomials and exponentials of polynomials, provided that no more than n of the D_j intersect at