

Two Examples of Non-trivial Essentially n -normal Operators

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1. Introduction. The purpose of this paper is to present two concrete operators which, hopefully, are illuminating examples of certain properties in the structure theory of operators on Hilbert space. In this section we describe the properties that these operators exhibit. We refrain from introducing the more technical definitions until Section 2. In Sections 4 and 5 we present the proofs that these operators enjoy the properties claimed. Section 3 contains a calculation of Ext of the various suspensions of the algebra of $k \times k$ complex matrices \mathbb{M}_k .

The first operator that we shall discuss is derived from consideration of the second suspension of \mathbb{M}_2 and demonstrates the torsion phenomenon first described in [7]. In particular, we construct an essentially 2-normal operator S , such that S is not a compact perturbation of a 2-normal operator, but such that $S \oplus S$ is. An example of such an operator has already been given in [7], but in that example we found it difficult to grasp intuitively why such a phenomenon had occurred. The operator S that we shall present is more easily written down and the reasons for $S \oplus S$ being a compact perturbation of a 2-normal operator are immediately apparent.

The second operator that we will consider answers a question raised in [13, Problem D] and [14] in the negative. An operator T is said to be (essentially, respectively) algebraically n -normal if the unital C^* -algebra generated by T , $C^*(T)(\pi C^*(T))$, respectively) satisfies a certain polynomial identity enjoyed by the $n \times n$ matrices [14, Definition 4] or equivalently [4] and [15], if every irreducible representation of $C^*(T)(\pi C^*(T))$, respectively) is at most n -dimensional. An operator T in $\mathcal{L}(\mathcal{H})$ is said to be (essentially) n -normal if there is a unitary transformation $U : \mathcal{H} \rightarrow \mathcal{H} \otimes \mathbb{C}^n$ such that UTU^* is an $n \times n$ matrix of operators, all of whose entries are (essentially) commuting, (essentially) normal elements of $\mathcal{L}(\mathcal{H})$. An operator which is (essentially) n -normal is (essentially) algebraically n -normal [14, Proposition 2].

It is well known [12] that if T is algebraically n -normal, then T is unitarily equivalent to

$$\sum_{k=1}^n \oplus T_k,$$