

Basically Scattered Vector Measures

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1. Examples. Throughout this paper Σ is a σ -field of subsets of a point set Ω and X is a real Banach space with dual X^* . If $F: \Sigma \rightarrow X$ is a (countably additive) vector measure, a set $E \in \Sigma$ is called F -null if F vanishes on every set $A \in \Sigma$ such that $A \subseteq E$. For economy of exposition we shall agree that a subset E of Ω is non- F -null if $E \in \Sigma$ and E is not F -null. A countably additive vector measure $F: \Sigma \rightarrow X$ is called *basically scattered* if for each disjoint sequence (E_n) of non- F -null sets the sequence $(F(E_n))$ is a basic sequence in X . Examples of basically scattered vector measures abound. Let (Ω, Σ, μ) be a finite non-atomic measure space and define $F: \Sigma \rightarrow L_1(\mu)$ by $F(E) = \chi_E$ for $E \in \Sigma$. Then F is a basically scattered vector measure of bounded variation. We shall show, in Section 2, that any non-atomic non-zero basically scattered vector measure of bounded variation is locally an isomorphic image of the measure F defined above. If we define $F: \Sigma \rightarrow L_p(\mu)$ ($1 < p < \infty$) by $F(E) = \chi_E$ for $E \in \Sigma$ we find that F is a non-atomic basically scattered vector measure. In the fourth section we shall show that if a basically scattered vector measure G has the property that $(G(E_n))$ is equivalent to the ℓ_p ($1 \leq p < \infty$) unit vector basis for each disjoint sequence (E_n) of non- F -null sets, then G is locally the isomorphic image of the measure F defined above. In the third section we shall see that if X is a Banach space that does not contain ℓ_∞^n 's uniformly and such that there is a non-atomic basically scattered vector measure with range in X , then X contains an infinite-dimensional Hilbertian subspace.

More examples of basically scattered vector measures are furnished by the symmetric non-atomic random measures of Urbanik and Woyczynski [16]. This follows easily from the fact that if f and g are independent symmetric random variables, then $E\|f\| \leq E\|f + g\|$. In fact many of the results of this paper can be proved for symmetric non-atomic random measures of finite expectations by the methods of [16].

Additional examples of basically scattered vector measures arise naturally in the theory of cyclic Banach spaces. If X is a cyclic Banach space then according to a theorem of Bade [1] (see [3, XVII. 3.10]) there is a spectral measure P on a σ -field Σ of subsets of a point set Ω such that for each $x \in X$, $F(E)x$ is a countably additive function of $E \in \Sigma$ such that there is an $x_0 \in X$ with the