

Initial and Final-Value Problems for Degenerate Parabolic Evolution Systems

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1. Introduction. We begin by showing that a Cauchy problem for the linear implicit evolution equation

$$(1.1) \quad \frac{d}{dt} (Mu(t)) + Lu(t) = f(t), \quad t > 0$$

is *continuously solvable*. By this we mean that for appropriate data there exists a (not necessarily unique) solution which depends continuously on the data with respect to certain seminorms. The non-negative and Hermitian operator M may be degenerate (*i.e.*, may vanish on non-zero vectors) and L is required to satisfy a sector condition. In Section 2 we show that this Cauchy problem is resolved by an analytic semigroup. Analogous results are obtained in Section 4 for a second-order system which is equivalent to the equation

$$(1.2) \quad \frac{d}{dt} \left(\frac{d}{dt} Cu(t) + Bu(t) \right) + Au(t) = f(t), \quad t > 0$$

when C and A are non-negative and Hermitian and B satisfies a sector condition. These results on continuous solvability of Cauchy problems for (1.1) and (1.2) are extensions of certain related results in [1]. Although we do obtain new applications to initial-boundary value problems for degenerate partial differential equations, the primary objective of the above presentation is to provide the background material for our treatment below of the corresponding final-value problems.

The method of *quasi-reversibility* (QR) will be used to approximate the solutions of final-value problems for (1.1) and (1.2) in the situation described above. The equations are *parabolic*, in the sense that they are resolved by analytic semigroups, so the final-value problems are necessarily ill-posed. In a QR-method one begins with final data at time $t = T > 0$ and solves an approximating equation backward in time to obtain corresponding initial data at time $t = 0$. Then the original equation is solved forward starting from the initial data. The general QR-method was introduced by Lattes and Lions [5]. We shall implement the QR-method in Section 3 where we approximate (1.1) by