

# *L<sup>p</sup>-Densities and Boundary Behaviors of Green Potentials*

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**1. Introduction.** Let  $u$  be a Green potential in the upper half space  $H \equiv \{x \in \mathbb{R}^n: x_n > 0\}$  ( $n \geq 2$ ) given by a mass distribution  $\mu$ , that is

$$u(x) = \int_H G(x, y) d\mu(y),$$

where  $G$  is the Green function on  $H$ . We assume without further comment that  $u \not\equiv +\infty$ , which is equivalent to

$$(1.1) \quad \int_H \frac{y_n}{1 + |y|^n} d\mu(y) < \infty.$$

It is well known that  $u$  has vertical limit ( $x_n \rightarrow 0^+$ ) zero almost everywhere on  $P \equiv \{x: x_n = 0\}$  but  $u$  need not have nontangential limit at any point on  $P$  even if  $\mu$  is absolutely continuous with respect to the Lebesgue measure (see [9], [11] and [13], the example in [13] needs to be modified).

In [2], Brelot introduced the concept of a “fine-limit” and in [10, Theorem 21], Naïm proved that  $u$  has fine-limit zero almost everywhere on  $P$ . Combining this with results of Deny [5] and Lelong-Ferrand [8] on fine-limit and capacities, we easily conclude the following:

**Theorem A.** *At almost every  $a \in P$ ,  $u$  has limit zero along all rays in  $H$  to  $a$  except a collection of rays which meet  $\{x: x_n = 1\}$  on a set of  $(n - 2)$ -capacity zero ( $n \geq 3$ ) or of logarithmic capacity zero ( $n = 2$ ).*

Suppose  $\mu$  is given by a density function  $\lambda$ ; an  $L^p$ -condition on  $\lambda$  guaranteeing the almost-everywhere existence of the non-tangential limit was given by Arsove and Huber [1]:

**Theorem B.** *Let  $D = \{x \in H: |x| < 1\}$  and  $p > n/2$ . Suppose  $d\mu(y) = \lambda(y)dy$  and  $\lambda$  satisfies*

$$\int_D \lambda^p(y) y_n^{2p-1} dy < \infty.$$